

Name: _____

Solve the following problems.

1. Find a bound on $R_1 \sin(x)$ which is valid for all x with $0 \leq x \leq 1$. Use this to show that there is a constant $C > 0$ so that for all integers $n \geq 1$, $\sin\left(\frac{1}{n}\right) \geq \frac{1}{n} - \frac{C}{n^2}$. Why does this show that the sum

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

diverges?

2. Find a bound on $R_1 \arctan(x)$ which is valid for all x with $0 \leq x \leq 1$ and use this to show that the sum

$$\sum_{n=1}^{\infty} \arctan\left(\frac{1}{n^2}\right)$$

converges.

3. For which values of x does the series $\sum_{n=1}^{\infty} \frac{x^n n!}{(n!)^2} e^{nx}$ converge? Justify your answer.
4. Find a bound on $R_n(\sin(x) + \cos(x))$ and use this to show that $T_n(\sin(x) + \cos(x))$ converges to $\sin(x) + \cos(x)$ as $n \rightarrow \infty$.
5. Find a bound on $R_n e^{2x}$ and use this to show that for every x , $T_n e^{2x}$ converges to e^{2x} as $n \rightarrow \infty$.
6. For which values of x is it true that $T_n \frac{1}{2-x}$ converges to $\frac{1}{2-x}$ as $n \rightarrow \infty$? Justify your answer.