

Name: \_\_\_\_\_

Solve the following problems.

1. Hasdrubal has designed a rocket. While proving mathematically that it won't explode, he used the approximation  $e^{\frac{1}{3}} \approx 1 + \frac{1}{3} + \frac{1}{3^2 2!} + \frac{1}{3^3 (3!)}$ . If this approximation is off by more than  $\frac{2}{4!}(\frac{1}{3})^4$  then the rocket might blow up. Convince Hasdrubal that it won't.
2. Find a bound for  $R_n^0 \sin(3x)$  and use this to show that  $T_n^0 \sin(3x) \rightarrow \sin(3x)$  for all  $x$  as  $n \rightarrow \infty$ .
3. Find a bound on  $|R_n \cos(x)|_{x=1}$  and use this information to find a decimal approximation of  $\cos(1)$  with an error of at most .1.

4. Find the Taylor series around zero for  $\cosh(2x) = \frac{1}{2} (e^{2x} + e^{-2x})$ .

5. Find the degree two Taylor polynomial around 0 of  $\frac{e^x}{1-x}$  without computing any derivatives.

6. Let  $f(x) = xe^{3x^2}$ . Compute  $f^{(2015)}(0)$  and  $f^{(2016)}(0)$ .