

1. Find the domain of the following functions.

(a)  $f(x, y) = \sqrt{4 - x^2 - y^2}$

(b)  $f(x, y) = \sqrt{4 + x^2 + y^2}$

(c)  $f(x, y) = \sqrt{y - x^2}$

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2. Consider the following three functions.

$$\gamma_1(t) = \begin{pmatrix} 1 + t^2 \\ t \end{pmatrix}, \quad \gamma_2(t) = \begin{pmatrix} 1 + t^3 \\ t^3 \end{pmatrix}, \quad \gamma_3(t) = \begin{pmatrix} 2 \cos t \\ 2 \sin t \end{pmatrix}.$$

(a) Which of the above functions traces out a curve that passes through  $(0, -1)$ ? There may be more than one. Justify your answer in each case.

(b) Which one traces out a line? Which one traces out a circle? Which one traces out a parabola?

(c) For each function, write a Cartesian equation (in the variables  $x, y$ ) which describes the curve that is traced out by that function.

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3. Consider the points  $A(1, 3, 1)$ ,  $B(2, 2, -1)$ , and  $C(2, 0, 3)$ .

(a) Find an equation for the plane containing the points  $A, B, C$ .

(b) Find the area of the triangle with vertices  $A, B, C$ .

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4. For each of the following, compute the path derivative  $(f \circ \gamma)'(t)$  at the given value of  $t$ .

(a)  $f(x, y) = x^2y$ ,  $\gamma(t) = \begin{pmatrix} t^2 \\ t \end{pmatrix}$ ,  $t = 1$ .

(b)  $f(x, y, z) = x + yz$ ,  $\gamma(t) = \begin{pmatrix} t \\ 2t \\ 3t \end{pmatrix}$ ,  $t = 2$ .

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5. Consider the parametric curve

$$\gamma(t) = \begin{pmatrix} \cos(2t) \\ \sin(2t) \\ (4\sqrt{2})t \end{pmatrix}.$$

- (a) Find the equation for the tangent line at the point  $(0, 1, \pi\sqrt{2})$ .
  - (b) Does the line from (a) intersect the  $xy$ -plane? If so, where?
  - (c) Find the speed and the acceleration vector at  $(0, 1, \pi\sqrt{2})$ .
  - (d) Find the linear acceleration and the angular acceleration at time  $t$ .
  - (e) Find the unit tangent vector and the unit normal vector at time  $t$ .
  - (f) Find the curvature at time  $t$ .
  - (g) Find the length of the part of the curve from  $(0, 1, \pi\sqrt{2})$  to  $(1, 0, 4\pi\sqrt{2})$ .
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6. Mark each of the following statements as true or false:

- (a) For motion in space, if the speed is constant, then the acceleration is always zero.
  - (b) For motion in space, if the velocity is constant, then the acceleration is always zero.
  - (c) For motion in space, the velocity is always perpendicular to the acceleration.
  - (d) If the velocity of a particle is always perpendicular to acceleration, then the speed of the particle must be constant.
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7. Consider the following parametric curves:

$$\mathbf{u}(t) = \begin{pmatrix} t \cos t \\ t \sin t \\ t^2 \end{pmatrix}, \quad \mathbf{v}(t) = \begin{pmatrix} \cos t \\ 2 \sin t \\ -1 - 3 \sin^2 t \end{pmatrix}, \quad \mathbf{w}(t) = \begin{pmatrix} e^t \cos(5t) \\ e^t \sin(5t) \\ e^t \end{pmatrix}.$$

- (a) Which of these curves lie on the surface  $z^2 = x^2 + y^2$ ?
  - (b) Which of these curves lie on the surface  $z = -x^2 - y^2$ ?
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8. Let  $f(x, y) = 2x^3 - \frac{1}{4}xy^2$ .

- (a) Find an equation for the tangent plane to the graph of  $f(x, y)$  at  $(1, 2, f(1, 2))$ .
  - (b) Estimate the value of  $f(0.95, 2.1)$ .
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9. Suppose that  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{w}$  are vectors in 3D space such that  $\|\mathbf{u}\| = 5$ ,  $\|\mathbf{w}\| = 2$ ,  $\mathbf{u} \cdot \mathbf{v} = 7$ , and  $\mathbf{u} \cdot \mathbf{w} = -3$ . Find each of the following quantities:

- (a)  $\mathbf{u} \cdot \mathbf{u}$

(b)  $\mathbf{w} \cdot 2\mathbf{u}$

(c)  $\mathbf{u} \cdot (\mathbf{v} - \mathbf{w})$

(d)  $(\mathbf{u} + \mathbf{w}) \cdot (\mathbf{u} - \mathbf{w})$