1. Show that if a curve has constant speed, then its acceleration is always perpendicular to its velocity. (Hint: Rewrite the condition "constant speed" in terms of the dot product, and differentiate.)

2. Show that if a particle always moves along a circle, then its velocity vector is always tangent to the circle. (Hint: Rewrite the condition "moves along a circle" in terms of the dot product. What does it mean to be tangent to a circle?)

3. Suppose
$$\mathbf{x}(t) = \begin{pmatrix} R\cos(\omega t) \\ R\sin(\omega t) \end{pmatrix}$$
where R and ω are positive constants.

   (a) Find $\|\mathbf{v}(t)\|$.

   (b) Write $\mathbf{a}(t)$ in terms of $\mathbf{x}(t)$.

   (c) Rewrite Newton's formula ($\mathbf{F} = m\mathbf{a}$) in terms of $\mathbf{x}(t)$, and find $\|\mathbf{F}\|$ in terms of $\|\mathbf{v}(t)\|$. You should end up with the familiar formula for centrifugal force.

4. Consider the curve $\mathbf{x}(t) = \begin{pmatrix} e^t \cos t \\ e^t \sin t \end{pmatrix}$.

   (a) Check that the points $(1,0)$ and $(-e^\pi, 0)$ are on this curve. (*What are the corresponding values of t?*)

   (b) Find the arc length between these two points.

   (c) Find the point on the curve that is exactly midway **along the curve** between these two points.

5. Consider the parametric curve given by:
$$\mathbf{x}(t) = \begin{pmatrix} e^t \cos t \\ e^t \sin t \\ e^t \end{pmatrix}$$

   (a) Find the arc length from $t = 0$ to $t = \pi$.

   (b) Parametrize the line that is tangent to $\mathbf{x}(t)$ at the point t_0.

(c) Find the point where the above tangent line intersects the xy-plane. Let $\mathbf{y}(t_0)$ be the position vector for this point. As t_0 varies, sketch the curve that $\mathbf{y}(t_0)$ traces.