

Quiz 7 RM Solutions

Please inform your TA if you find any errors in the quiz solutions.

1. (4 points) True or false? Please circle your answer.

• $T_{\infty} \cos(2x^3) = \sum_{n=0}^{\infty} \frac{2^{2n+1} x^{6n+3}}{(2n+1)!}$ **True** **False**

• $T_{2015} \sum_{n=0}^{1848} 3x^n = \sum_{n=0}^{1848} 3x^n$ **True** **False**

• $f^{(42)}(0) = 0$, where $f(x) = \cos(x^2)$ **True** **False**

• $T_3 \frac{e^x}{1+x} = 1 + x + x^2 + x^3$ **True** **False**

Solution:

1. False
2. True
3. True

4. False

2. (6 points) Show that $|\frac{1}{2} - \frac{1}{e}| \leq \frac{1}{3!}$. Hint: $\frac{1}{2} = 1 + \frac{-1}{1} + \frac{(-1)^2}{2!}$ is the approximation obtained from the second Taylor polynomial of e^x .

Solution: Let $f(x) = e^x$. Taylor's theorem tells us that there is a ξ with $-1 \leq \xi \leq 0$ such that

$$\frac{1}{2} - \frac{1}{e} = \frac{f^{(3)}(\xi)}{3!}(-1)^3$$

Now note $f^{(3)}(\xi) = e^\xi \leq 1$. Therefore

$$|\frac{1}{2} - \frac{1}{e}| \leq \frac{1}{3!}$$