MATH 222(1,2,4) Fall 2015
Quiz 2RMSolutions

Please inform your TA if you find any errors in the quiz solutions.

1. (*4 points*)

1. (2 *points*) Compute A and B so that

$$\frac{1}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}.$$

Solution:

$$\begin{aligned}\frac{1}{x(x-3)} &= \frac{A}{x} + \frac{B}{x-3} \\ 1 &= A(x-3) + Bx\end{aligned}$$

Now, plugging in $x = 0$ lets us solve for $A = -1/3$, and plugging in $x = 3$ gives that $B = 1/3$. Alternatively, we can rewrite the previous line as

$$0x + 1 = (A+B)x - 3A,$$

and equate the coefficients of x and 1 to obtain $A + B = 0$ and $-3A = 1$.

2. (2 *points*) Write $\frac{x^2+4x+1}{x-3}$ as the sum of a polynomial and a proper rational function.

Solution:

Since the degree of the numerator is larger than the degree of the denominator, this is not a proper rational function. We will use polynomial long division:

$$\begin{array}{r|l} & \quad x \quad + 7 \\ \hline x-3) & x^2 + 4x \quad + 1 \\ & -x^2 + 3x \\ \hline & \qquad 7x \quad + 1 \\ & \qquad -7x + 21 \\ \hline & \qquad\qquad 22 \end{array}$$

Therefore,

$$\frac{x^2+4x+1}{x-3} = x + 7 + \frac{22}{x-3}.$$

2. (*6 points*)

1. (3 *points*) The following is the reduction formula for $I_n = \int \cos^n x\,dx$.

$$I_n = \frac{1}{n}\sin x\cos^{n-1}x + \frac{n-1}{n}I_{n-2}$$

   (a) Use the given reduction formula to evaluate $\int_0^\pi \cos^4 x\,dx$.

   (b) Verify the reduction formula.

Solution:

(a) First notice that the reduction formula relates $\int \cos^n x\,dx$ to $\int \cos^{n-2} x\,dx$. Since the power of cosine is even, our base case is $n = 0$, and we compute that directly.

$$\begin{aligned} I_0 &= \int_0^\pi dx \\ &= x|_0^\pi \\ &= \pi - 0 \\ &= \pi \end{aligned}$$

Next we use the reduction formula to find I_2.

$$\begin{aligned} I_2 &= \frac{1}{2}\sin x\cos x|_0^\pi + \frac{1}{2}(\pi) \\ &= \frac{1}{2}\sin(\pi)\cos(\pi) + \frac{1}{2}\sin(0)\cos(0) + \frac{\pi}{2} \\ &= \frac{\pi}{2} \end{aligned}$$

Finally, we use the reduction formula a second time to compute I_4.

$$\begin{aligned} I_4 &= \frac{1}{4}\sin x\cos^3 x|_0^\pi + \frac{3}{4}\cdot\frac{\pi}{2} \\ &= \frac{1}{4}\sin(\pi)\cos^3(\pi) + \frac{1}{4}\sin^3(0)\cos(0) + \frac{3\pi}{8} \\ &= \frac{3\pi}{8} \end{aligned}$$

(b)

$$\begin{aligned}
I_n &= \int \cos^n x \, dx \\
&= \int \underbrace{\cos^{n-1} x}_{\text{F(x)}} \underbrace{\cos x \, dx}_{\text{G'(x)dx}} \\
&= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \sin^2 x \, dx \\
&= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x (1 - \cos^2 x) \, dx \\
&= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^2 x \, dx \\
I_n &= \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n \\
nI_n &= \cos^{n-1} x \sin x + (n-1) I_{n-2} \\
I_n &= \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}
\end{aligned}$$