FINAL EXAM
CALCULUS III
MONDAY DECEMBER 16, 2013

- The use of class notes, book, formulae sheet, calculator is not permitted.
- In order to get full credit, you **must** show all your work.
- Please write solution to a problem in the space provided.
- You have **two hours and fifty minutes**.
- Do not forget to write your name and UNI in the space provided below.

Print UNI ____________________

Print Name ____________________

Section ____________________

For Grader's use only:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**
___/16	___/10	___/12	___/8	___/10	___/10	___/16	___/8	___/90

Problem 1 (16 points)

(a) Let $f(x, y)$ be a function of two variables such that the following directional derivatives are known at $(0, 0)$.

$$D_{\frac{\langle 1,1\rangle}{\sqrt{2}}} f(0,0) = -\sqrt{2} \qquad D_{\frac{\langle 2,-1\rangle}{\sqrt{5}}} f(0,0) = \sqrt{5}$$

Compute the gradient of $f(x, y)$ at $(0, 0)$.

(b) Prove that

$$\lim_{(x,y)\to(0,0)} \frac{2xy^5}{x^2 + y^8} = 0$$

(c) Find all (complex) solutions to the following equation:

$$z^3 - z^2 + z - 1 = 0$$

(d) Compute the three cuberoots of $z = -1 + \sqrt{3}\, i$.

Problem 2 (10 points) The plane $4x - 3y + 8z = 5$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse. Use Lagrange multipliers to find the highest and lowest points on this ellipse. (Highest/lowest = largest/smallest z–coordinate).

Problem 3 (12 points) Let $F(x, y, z) = \ln(1 + z(x^2 - y^3))$.

(a) Compute the differential of F.

(b) Find the direction in which F increases most rapidly at $(1, 1, 3)$. What is the fastest rate of increase?

(c) Use linear approximation at $(1, 1, 3)$ to find the approximate value of $F(0.95, 0.95, 3.01)$.

Problem 4 (8 points) A projectile is fired from a point 80 m above the ground, with an initial speed of 60 m/s at an angle of $30°$. (use acceleration due to gravity $g = 10\ m/s^2$).

(a) Write the position $\vec{\mathbf{r}}(t)$ and velocity $\vec{\mathbf{v}}(t)$ at time t.

(b) Where does the projectile hit the ground?

Problem 5 (10 points) Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 2y$ on the region bounded by the triangle with vertices $(2, 0), (0, 2)$ and $(-2, 0)$.

Problem 6 (10 points) Find the critical points of $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$ and use the second derivative test to classify them as local maxima, local minima or saddle point.

Problem 7 (16 points)

(a) Find the parametric equations describing the intersection of the following two surfaces

$$z = \sqrt{x^2 + y^2} \qquad \text{and} \qquad z = 1 + y$$

(b) Find the length of the following parametric curve:

$$\vec{\mathbf{r}}(t) = \left\langle 9t, (2t)^{3/2}, \frac{t^2}{2} \right\rangle \qquad 0 \le t \le 2$$

(c) Find the rate of change of the volume of a rectangular box, if its length and width are increasing at a rate of 1 cm/s while its height is decreasing at a rate of 2 cm/s, when its length and width are 20 cm and its height is 10 cm.

(d) Write the equation of the tangent plane to $xy + yz + xz = 3$ at the point $(1, 1, 1)$.

Problem 8 (8 points) Let $\vec{\mathbf{r}}(t) = \langle t, 2t, t^2 \rangle$ be the parametric curve describing the path of a particle.

(a) Compute the tangential and normal components of its acceleration.

(b) Find the curvature at time $t = 2$.