PRACTICE FINAL
CALCULUS III

(1)
- Find the volume of the parallelopiped formed by:

$$\vec{\mathbf{u}} = \langle 1, 0, 2 \rangle \qquad \vec{\mathbf{v}} = \langle 2, -1, 0 \rangle \qquad \vec{\mathbf{w}} = \langle 4, 1, 1 \rangle$$

- Find the parametric equations describing the tangent line to the following parametric curve, at $(2, -4, 3)$

$$\vec{\mathbf{r}}(t) = \left\langle \sqrt{2t}, 4 - t^3, t^2 - 1 \right\rangle$$

- Write $w = \dfrac{1 + \sqrt{3}\, i}{1 - \sqrt{3}\, i}$ in $a + b\, i$ form.
- Find the length of the following curve

$$\vec{\mathbf{r}}(t) = \left\langle 12t, 8t^{3/2}, 3t^2 \right\rangle \quad 0 \le t \le 1$$

- Compute the following limit, or prove that it doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{5x \sin^2(y)}{x^2 + y^4}$$

(2) Let $f(x, y) = \ln(1 + x^2 + y^2)$.
- Compute f_{xx} and f_{xy}.
- Write the unit vector along which f is increasing fastest at $x = y = 1$.
- What is the rate of change of f at $(1, 1)$ in the direction of $\langle 1, -2 \rangle$.

(3) Find all critical points of $f(x, y) = 2x^3 + y^3 - 5xy$ and classify them as local minimum, local maximum, saddle point.

(4) Find the absolute maximum and minimum values of $f(x, y) = xy - 5x^2 + 3$ on the domain D bounded by x–axis, $x = 2$ and $y = x^3$.

(5) Let C be the curve of intersection of the following two surfaces

$$x^2 + y^2 = 1 \qquad \text{and} \qquad z = 3 - 2x^2 - 4y^2$$

Find points on C which are closest to and furthest from the origin.

(6) A projectile is fired with an initial speed of 100 m/s at an angle of 60°.
- Write the position and velocity of the projectile as a function of t.
- At what times is the projectile at a height three quarters of its maximum height.

(7) Assume that z is implicitly defined as a function of x, y by

$$\cos(yz) + x^2 z = 9$$

If at $x = 2, y = 0, z = 2$, the value of x starts increasing at a rate of 1 unit per second, and the value of y starts decreasing at a rate of 2 units per second, compute the rate of change of z.

(8) Let $\overrightarrow{\mathbf{r}}(t)$ be a parametric curve. Prove that

$$\frac{d}{dt}\left(\frac{\overrightarrow{\mathbf{r}}(t)}{|\overrightarrow{\mathbf{r}}(t)|}\right) = \frac{1}{|\overrightarrow{\mathbf{r}}(t)|}\left(\overrightarrow{\mathbf{r}}'(t) - \mathrm{Proj}_{\overrightarrow{\mathbf{r}}(t)}(\overrightarrow{\mathbf{r}}'(t)\right)$$

(9) Find the distance between the point $(1, 3, 2)$ and the line

$$\frac{x-5}{4} = y = \frac{z-1}{2}$$