Name: **Practice Final**

1. Consider the line $\vec{r}(t) = \langle 3+2t,\ 2-t,\ 6\rangle$.

(a) Find symmetric equations for this line.

(b) Find the point where the line $\vec{r}(t)$ intersects the surface $z = x + y^2$.

2. Consider the two lines $\vec{r}(t) = \langle 1+t,\ 2+t,\ 3-t\rangle$ and $\vec{s}(t) = \langle t,\ -1+3t,\ 1+2t\rangle$.

(a) Find an equation for the plane that contains the two lines $\vec{r}(t)$ and $\vec{s}(t)$.

(b) Find the distance from this plane to the point $P = (1, 2, 4)$.

3. Find the distance between the plane $x + 2y - z = 4$ and the plane $-3x - 6y + 3z = 9$. Then find the parametric equation of the line that passes through the point $(1, 1, 1)$ and is perpendicular to both planes.

4. Identify the surfaces $4y^2 + z^2 - x - 16y - 4z + 10 = 0$ and $2x^2 - y^2 + z^2 + 4x - 4y - 6z + 7 = 0$.

5. Consider the curve

$$\vec{r}(t) = \langle 12t,\ 8t^{3/2},\ 3t^2\rangle,$$

with $0 \leqslant t \leqslant 1$.

(a) Find the velocity $\vec{r}\,'(1)$ and acceleration $\vec{r}\,''(1)$ at time $t = 1$.

(b) Find the total length of the curve.

6. A surface in $\mathbb{R}^3$ is described by the equation

$$e^{xyz} = z\ln(xy) + e^2.$$

(a) Find an equation for the tangent plane to the surface at the point $(1, 1, 2)$.

(b) Find a parametrization for the line that passes through $(1, 1, 2)$ and is perpendicular to the tangent plane to the surface (i.e. the normal line).

(c) Find the point where the normal line intersects the xz-plane.

7. Consider the surface S in $\mathbb{R}^3$ given by the equation $z = e^x \cos(xy^2)$.

(a) Find an equation for the tangent plane to S at the point $(0, 0, 1)$.

(b) Use linear approximation to estimate the value of $e^{0.001} \cdot \cos(0.001 \cdot (0.01)^2)$.

8. Find a point on the surface $z = -\frac{e^x}{2} + 3y^2$ where the tangent plane is parallel to the plane $x - y + 2z = 3$.

9. Find the absolute maximum and minimum of $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

10. Find the absolute maximum and minimum values of $f(x, y) = x^2 + xy$ on the region D in $\mathbb{R}^2$ that is enclosed by the parabolas $y = x^2 - 1$ and $y = 1 - x^2$. Be sure to specify all the points where the maximum/minimum are attained.

11. Suppose the sum of three positive real numbers is 9. What is the maximum possible value of their product? (Hint: use Lagrange multipliers.)