

**Midterm 1 Sample**  
**Math UN1101: Calculus III, Section 2**  
**Spring 2019**  
**Instructor: Linh Truong**

Name: \_\_\_\_\_

Instructions:

- Print your name in the space above.
- Show your reasoning and intermediate computations.
- You have 75 minutes.
- No notes, books, calculators or any other electronic devices are allowed.
- Write answers in the space provided. If you need extra space, use the backs of pages.

1. Let  $\vec{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\vec{b} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  and  $\vec{c} = \mathbf{i} + \mathbf{k}$ . Compute the followings:

(a)  $\vec{a} + \vec{b} - \vec{c} =$

(b)  $\vec{a} \times \vec{c} =$

(c) Volume of the parallelepiped determined by  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .

(d)  $\text{Proj}_{\vec{b}}\vec{c} =$

2. (a) Identify the surface in  $\mathbb{R}^3$  described in spherical coordinates by  $\phi = 3\pi/4$ .

(b) Find an equation for the surface in rectangular coordinates.

3. Let  $\mathcal{P}_1$  be the plane  $x + y + z = 1$ , and  $\mathcal{P}_2$  be the plane  $x - y + z = 1$ .
- (a) Find cosine of the angle between  $\mathcal{P}_1$  and  $\mathcal{P}_2$ .
  
  
  
  
  
  
  
  
  
  
  - (b) Find symmetric equation for the line of intersection of  $\mathcal{P}_1$  and  $\mathcal{P}_2$ .
  
  
  
  
  
  
  
  
  
  
  - (c) The plane  $\mathcal{P} : 2y - 2x - 2z = 3$  is parallel to either  $\mathcal{P}_1$  or  $\mathcal{P}_2$ . Which one?
  
  
  
  
  
  
  
  
  
  
  - (d) Compute the distance between the parallel plane to  $\mathcal{P}$ , from part (c), and  $\mathcal{P}$ .

4. Let  $L_1$  and  $L_2$  be the lines:

$$L_1 : \quad x = 1 + t, \quad y = 1 + 6t, \quad z = 2t$$

$$L_2 : \quad x = 1 + 2s, \quad y = 5 + 15s, \quad z = 6s - 2.$$

(a) The line  $x - 2 = -\frac{1-y}{6} = \frac{z-2}{2}$  is parallel to either  $L_1$  or  $L_2$ . Which one?

(b) The plane  $2x + 12y + 4z = 5$  is orthogonal to either  $L_1$  or  $L_2$ . Which one?

(c) Show that  $L_1$  and  $L_2$  are skew lines.

(d) Find an equation for a plane containing  $L_1$  and which doesn't intersect  $L_2$ .

5. (a) Describe the trace of the quadric surface

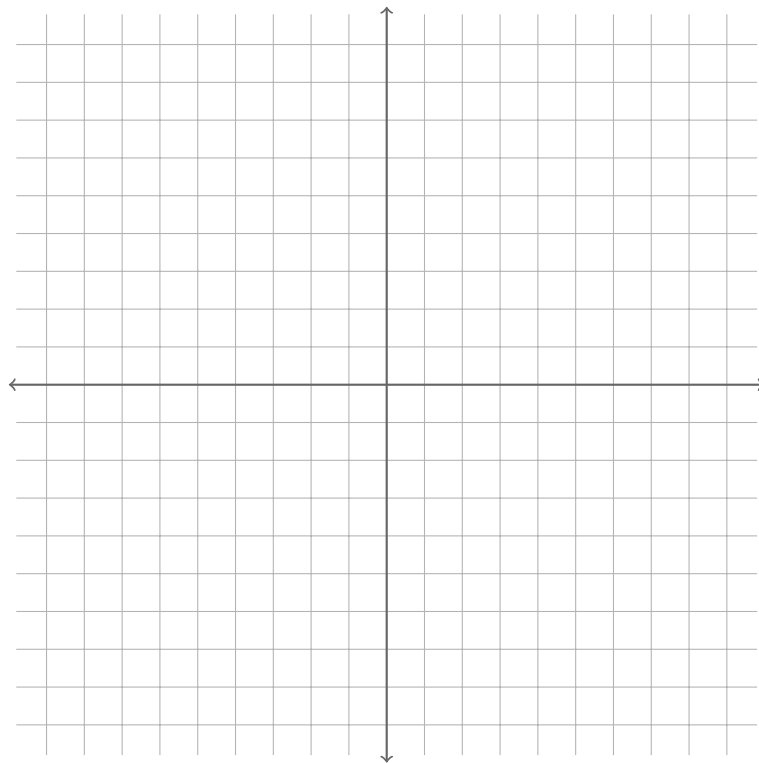
$$z = x^2 + 2y^2$$

in the plane  $z = 6$ .

- (b) Sketch the trace of the quadric surface

$$z = x^2 + 2y^2$$

in the  $yz$ -plane.



(c) Use the previous parts to sketch the quadric surface

$$z = x^2 + 2y^2.$$