PRACTICE MID TERM I
CALCULUS III

- The use of class notes, book, formulae sheet, calculator is not permitted.
- In order to get full credit, you **must** show all your work.
- Each solution must have a clearly labeled problem number and start at the top of a new page.
- You have **one hour and fifteen minutes**.
- Do not forget to write your name and UNI in the space provided below and on the notebook provided.

(1) Write the parametric equations describing the line
- lying on the plane $\mathbb{P} : x - 2y + z = 4$
- passing through the intersection of the plane $\mathbb{P}$ with the line $L : \vec{r} = \langle 1 + t, 1 - t, 2t \rangle$, and
- perpendicular to line L.

(2) Let $A = (1, 0, 1)$, $B = (3, 1, 0)$ and $C = (3, 2, 2)$ and $D = (-2, -2, 1)$ be four points in $\mathbb{R}^3$.
(a) Find the volume of the parallelopiped formed by edges AB, AC and AD.
(b) Find the coordinates of the point E opposite to A in this parallelopiped.
(c) Find the angle $\angle EAB$.

(3) Consider the plane $\mathbb{P} : x + y - 3z = 1$ and a point $P = (2, 1, 0)$.
(a) Find the parametric equations describing the line through P and perpendicular to the plane $\mathbb{P}$.
(b) Find the coordinates of the point Q where this line meets the plane $\mathbb{P}$.
(c) Compute the length of the line segment PQ.
(d) Compute the distance between the point P and the plane $\mathbb{P}$ directly (using the formula) and verify your answer from part (c).

(4) True/False. Justify your answer with a proof if true, or a counterexample if false.
(a) If $\vec{v} \cdot \vec{w} = 0$ then either $\vec{v} = 0$ or $\vec{w} = 0$.
(b) If $\vec{v} \times \vec{w} = 0$ then $\vec{v}$ and $\vec{w}$ must be parallel.
(c) A pair of lines is either parallel or intersect in a point.
(d) $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$.
(e) $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$.

(5) Consider the equation $z = ax^2 + y^2$.
(a) Sketch the traces for $a = -1, 0, 1$.
(b) Sketch the surface for $a = -1, 0, 1$.
(c) Describe how the surface changes when a approaches 0 from the left and from the right.

(6) Find the equation of the plane which contains the following two parallel lines:

$$x = 2 - t \qquad y = 3 + 2t \qquad z = 1 + t$$

$$1 - x = \frac{y - 4}{2} = z - 3$$

(7) Let $\vec{v}$ be a non–zero vector.
- Prove that $\text{Proj}_{\vec{v}}(\vec{u}) = 0$ if, and only if $\vec{u}$ is orthogonal to $\vec{v}$.
- Prove that $\text{Proj}_{\vec{v}}(\vec{u}) = \vec{u}$ if, and only if $\vec{u}$ is parallel to $\vec{v}$.