

# Calculus III, Midterm Exam 1

Calculus III, Math V1201, Section 001, Spring 2018

Instructor: Ilya Kofman

Date: February 14, 2018

Name: \_\_\_\_\_

Show all work. To receive full credit, you must justify your answers.

Use the back side of the page if you need more space to do a problem.

No calculators or electronic devices of any type are allowed.

<b>1</b>	<b>16</b>	
<b>2</b>	<b>20</b>	
<b>3</b>	<b>20</b>	
<b>4</b>	<b>20</b>	
<b>5</b>	<b>24</b>	
<b>Total</b>	<b>100</b>	

**Problem 1.**

- (a) Find the equation of the plane through  $P(1, 1, -3)$ ,  $Q(0, 2, -1)$  and  $R(1, 3, -2)$ .
- (b) Find the line of intersection between planes  $x - 2y + z = 1$  and  $3x + 2y + z = 5$ .

**Problem 2.** Let  $\ell$  be the line  $\frac{1}{3}(x - 1) = y + 2 = \frac{1}{2}(z - 4)$ , and let  $P(-2, 0, 1)$  be a point.

- (a) Find the equation of a plane containing the line  $\ell$  and passing through  $P$ .
- (b) Are  $\ell$  and the line  $r(t) = (-3 + t, 1 - t, t)$  skew, parallel or intersecting? Justify.

**Problem 3.** Consider the plane  $2x - 3y - z = -7$ . Let  $P(7, -2, -1)$  be a point.

- (a) Find a parametric equation of the line through  $P$  orthogonal to the plane.
- (b) Find the point  $Q$  on the plane closest to  $P$ .
- (c) Point  $R(0, 0, 7)$  is on this plane, which has normal  $\vec{n}$ . Compute  $\vec{v} = \text{proj}_{\vec{n}}(\overrightarrow{RP})$ .
- (d) How is  $\vec{v}$  related to your answer in part (b)? Justify.

**Problem 4.** For each of the following spheres, give its equation in Cartesian coordinates and identify its center and radius.

(a)  $S_1$  is given in spherical coordinates by  $\rho = \sin \theta \sin \phi$ .

(b) For  $A(0, 0, 1)$  and  $B(0, 2, 3)$ ,  $S_2$  is the set of points  $P(x, y, z)$  such that  $\overrightarrow{AP} \perp \overrightarrow{BP}$ .

**Problem 5.** Identify the following surfaces. Sketch each trace in the space provided. For each sketch, axes matter (e.g., major axis for ellipse), but you do not need to label vertices. Then sketch OR name precisely each surface.

Equation	$xy$ -trace	$yz$ -trace	$xz$ -trace	Surface
$z = 7x^2 - 3y^2$				
$5x^2 + y^2 - 2z^2 = 10$				
$x^2 - 4y^2 - z^2 = 4$				