MATH UN1201 **Midterm Exam #2** APRIL 1, 2020

You have until 11:59pm on Thursday, April 9, to complete the exam. Your completed exam must be uploaded to Gradescope by this time. You may use any resources for the exam, including the textbook, your notes, the internet, and a calculator, but you MAY NOT ask for help, communicate or discuss the exam with any person during the test (this includes posting on online forums). Sign the academic honesty statement at the bottom of the page indicating you understand and agree to these restrictions. If you are using your own paper, copy this statement in its entirety and sign beneath it.

If you have technical difficulties uploading your completed exam to Gradescope, email me your completed exam before the deadline. This will be a timestamp that you have completed the exam on time, and you can continue to troubleshoot the upload process.
Instructions:

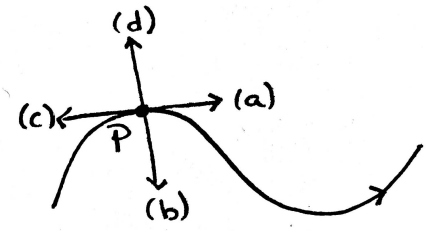
- Complete all problems and put a BOX around your final answer. You will lose credit if it is not clear what your final answer is.
- You must show your work or justify your answer for all problems, except where otherwise stated. *Answers without work or justification will receive no credit (even if they are correct).*
- If you use your own paper, do not put more than ONE problem on a page (multiple parts of a single problem on a page are acceptable).
- Keep your work organized (especially if you work on your own paper). It must be clear which work should be graded and which work goes with which problem.
- If you need more space, use the blank pages at the beginning and end of the exam, or use additional paper of your own. If you want me to grade work done any of those pages, clearly indicate this next to the appropriate problem.
- When writing vectors, you MUST use an arrow (in your work and in your answers). Failure to do so will result in loss of points.
- Leave your answers in an *exact* form (i.e., do not use decimal approximations). Simplify the arithmetic in your answers, but you do not have to simplify radicals or clear radicals from the denominators.

Academic Honesty Statement: I affirm that I will not plagiarize, use unauthorized materials, or give or receive illegitimate help on assignments, papers, or examinations. I will also uphold equity and honesty in the evaluation of my work and the work of others. I do so to sustain a community built around this Code of Honor.

NAME_____

Blank page.

1. (a) *(2 points each)* The following diagram shows the path $\vec{r}(t)$ of a particle moving to the right. All of the vectors shown have length one.



i. Choose the letters corresponding to vectors which could represent the unit tangent vector, $\vec{T}$ and the unit normal vector, $\vec{N}$, of $\vec{r}(t)$ at the point P.

$\vec{T}$: _____ $\vec{N}$: _____

ii. Assume that the curve lies in the page. Does $\vec{B}$ point into the page or out of the page?

iii. Draw a possible acceleration vector, $\vec{a}$, to $\vec{r}(t)$ at the point P on the diagram. [1]

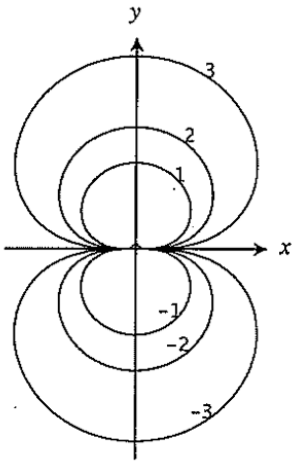
(b) *(2 points each)* Consider the equation $Ax^2 + By^2 + Cz^2 = D$. Fill in each blank with a single number that makes the statement correct.

i. If $A =$ _____, $B =$ _____, $C =$ _____, and $D =$ _____, then the graph of the equation is a hyperboloid of two sheets whose axis is the x–axis.

ii. If $A =$ _____, $B =$ _____, $C =$ _____, and $D =$ _____, then the graph of the equation is a (double) cone whose axis is the y–axis.

[1] If you are using your own paper, carefully copy the diagram and then draw $\vec{a}$

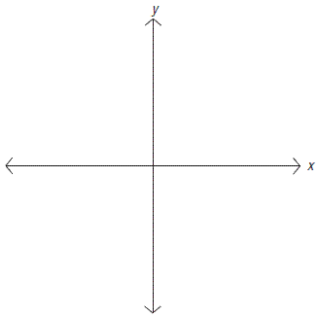
(c) *(3 points)* A function $f(x, y)$ is defined for $(x, y) \neq (0, 0)$. Is $\lim_{(x,y)\to(0,0)} f(x, y)$ likely to exist if f has the contour diagram shown below? Explain your answer.



2. Consider the function

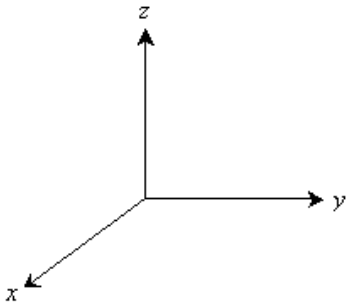
$$f(x,y) = \sqrt{16 - 4x^2 - \frac{16y^2}{9}}.$$

(a) *(5 points)* Find and sketch the domain of f on the axes provided. Label the x– and y–intercepts of your sketch.



(b) *(4 points)* Find the range of f.

(c) *(3 points)* Sketch the graph $z = f(x,y)$ on the axes provided. Label the x–, y–, and z–intercepts of your sketch.



3. *(6 points)* Find the length of the following curve from $t = 0$ to $t = 10$:

$$\vec{r}(t) = \left\langle 5t, \frac{4\sqrt{5}}{3}t^{3/2}, t^2 \right\rangle$$

4. *(10 points)* An object's acceleration is given by

$$\vec{a}(t) = \left\langle e^{2t}, 2(t+1)^{-2}, -t \right\rangle .$$

Find a vector equation $\vec{r}(t)$ for the position of the object at time t if the object's initial velocity is $\langle 1/2, -2, 1 \rangle$ and its initial position is $(0, 0, 0)$.

5. Consider the curve

$$\vec{r}(t) = \langle 2t, t^2, t^3 \rangle.$$

(a) *(5 points)* Find the equation of the tangent line L to $\vec{r}(t)$ at the point $(2, 1, 1)$.

(b) *(5 points)* Find the equation of the osculating plane $\mathbb{P}$ to $\vec{r}(t)$ at the point $(2, 1, 1)$.

(c) *(2 points)* How are the line L and the plane $\mathbb{P}$ related?

Recall that

$$\vec{r}(t) = \langle 2t, t^2, t^3 \rangle.$$

(d) *(5 points)* Find the curvature of $\vec{r}(t)$ at the point $(0, 0, 0)$.

(e) *(4 points)* Find the tangential and normal components of acceleration at $(0, 0, 0)$.

6. *(5 points each)* For each, find the limit or show that it does not exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x(e^{xy}-1)}{\sqrt{x^2+y^2}}$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^6 + y^3}$$

(c)

$$\lim_{(x,y)\to(0,0)} x\ln(x^2+y^2)$$

7. *(7 points)* Determine where the following function is continuous.

$$f(x,y) = \begin{cases} \frac{x^4-4y^2}{x^2+2y} & \text{if } (x,y) \neq (0,0) \\ 2 & \text{if } (x,y) = (0,0) \end{cases}$$

8. *(2 points each)* Determine if each statement is true or false, and write the (entire) word TRUE or FALSE in the space provided. You do not need to justify your answers.

(a) If $\vec{r}(t)$ is a differentiable function, then $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\vec{r}(t) \times \vec{r}(t)\right) = 2(\vec{r}\,'(t) \times \vec{r}(t))$

(b) If $\vec{r}(t)$ is a differentiable function, then $\dfrac{\mathrm{d}}{\mathrm{d}t}|\vec{r}(t)| = |\vec{r}\,'(t)|$.

(c) If a particle is moving along a circle, then the unit normal vector at any point always points towards the origin.

(d) If the tangential component of the acceleration of a particle is zero, then the speed of the particle is constant.

(e) If $f(x, y)$ approaches 1 as (x, y) approaches $(0, 0)$ along the x–axis, then $\lim_{(x,y)\to(0,0)} f(x, y) = 1$.

(f) The two level curves $f(x, y) = 3$ and $f(x, y) = 5$ of a function $f(x, y) = 2x^2 + 5y^2$ do not intersect at any point in the plane.

(g) If $f(x_0, y_0) = f(x_1, y_1)$, then $x_0 = x_1$ and $y_0 = y_1$.

Blank page.

Blank page.