The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, & Snider, Linear Algebra and Differential Equations. Note that the section numbers and problem numbers ARE NOT the same as in Lay, Linear Algebra.

Exercises 6.7: 9, 11, 13, 14, 25, 26

Exercises 6.2: 29, 30, 31

Exercises 6.3: 24

Exercises 6.4: 7, 15, 16, 19, 20, 22

Exercises 6.5: 15, 19, 20, 24, 25

Additional Problems:

1. Let A be an $m \times n$ matrix. Show that $\text{Row}(A)$ is orthogonal to $\text{Nul}(A)$ (with respect to the standard inner product on $\mathbb{R}^n$).

2. Show that the QR factorization of a nonsingular square matrix A is unique up to signs. More precisely, if

$$A = Q_1 R_1 = Q_2 R_2$$

are two such factorizations, show that there is a diagonal matrix D with diagonal entries ± 1 such that

$$Q_2 = Q_1 D, \quad R_2 = R_1 D.$$

(Note that $D^2 = I$, so this will give $Q_2 R_2 = Q_1 R_1$, as it should.) [1]

3. Let Q_1 and Q_2 be $m \times n$ matrices, each with orthonormal columns. Prove that if $\text{Col}(Q_1) = \text{Col}(Q_2)$, then $Q_1^T Q_2$ is an orthogonal matrix.

[1] Hint: First rearrange the equation $Q_1 R_1 = Q_2 R_2$ to get an identity equating an orthogonal matrix with an upper triangular matrix. Then show that if a matrix is both orthogonal and upper triangular, it must be diagonal, with diagonal entries ± 1.

4. Apply the Cauchy-Schwartz inequality for general inner product spaces to prove that for every continuous function $f\colon [-1,1] \to \mathbb{R}$, we have

$$\left(\frac{1}{2}\int_{-1}^{1} f(x)\,dx\right)^2 \le \left(\frac{1}{2}\int_{-1}^{1} f(x)^2\,dx\right).$$

In other words, the square of the average value of $f(x)$ on $[-1,1]$ is less than or equal to the average value of $f(x)^2$.