1. Find the Fourier series for each of the given function.

   (a) $f(x) = \begin{cases} 1, & -L \leq x < 0, \\ 0, & 0 \leq x < L; \end{cases} \qquad f(x+2L) = f(x).$

   (b) $f(x) = \begin{cases} x + L, & -L \leq x < 0, \\ L, & 0 \leq x < L; \end{cases} \qquad f(x+2L) = f(x).$

2. Let $f(x) = \begin{cases} -1, & -1 \leq x < 0, \\ 1, & 0 \leq x < 1, \end{cases}$ and assume f is periodically extended outside the interval $[-1, 1]$. Find the Fourier series for the extended function.

3. Find the Fourier sine series, with period 4, for $f(x) = \begin{cases} x, & 0 \leq x \leq 1, \\ 1, & 1 \leq x < 2. \end{cases}$

4. For each, determine whether the method of separation of variables can be used to replace the given partial differential equation by a pair of ordinary differential equations. If so, find the equations.

   (a) $tu_{xx} + xu_t = 0$

   (b) $u_{xx} + u_{xt} + u_t = 0$

5. Consider the conduction of heat in a rod 40 cm in length whose ends are maintained at 0°C for all $t > 0$. For each, find an expression for the temperature $u(x,t)$ if the initial temperature distribution in the rod is the given function. Suppose that $\alpha^2 = 1$.

   (a) $u(x, 0) = 50, \qquad 0 < x < 40$

   (b) $u(x, 0) = x, \qquad 0 < x < 40$

6. Consider the equation
$$au_{xx} - bu_t + cu = 0,$$
   where a, b, c are constants.

   (a) Let $u(x,t) = e^{\delta t} w(x,t)$, where δ is a constant, and find the corresponding pair of partial differential equation for w.

   (b) If $b \neq 0$, so that δ can be chosen so that the partial differential equation found in part (a) has no term in w. Thus, by a change of dependent variable, it is possible to reduce the original equation tot he heat equation.

7. The heat conduction equation in two space dimensions is

$$\alpha^2(u_{xx} + u_{yy}) = u_t.$$

Assuming that $u(x, y, t) = X(x)Y(y)T(t)$, find ordinary differential equations satisfied by $X(t)$, $Y(t)$, and $T(t)$.