Corrected Solution to ③:

First, we will find $\operatorname{irr}(\sqrt{2}+\sqrt{5}, \mathbb{Q})$:

$$x = \sqrt{2}+\sqrt{5}$$
$$x^2 = 2 + 2\sqrt{10} + 5$$
$$x^2 - 7 = 2\sqrt{10}$$
$$x^4 - 14x^2 + 49 = 40$$
$$x^4 - 14x^2 + 9 = 0$$

$f(x) = x^4 - 14x^2 + 9$ has no linear factors (because $\pm 1, \pm 3, \pm 9$ are not zeros of $f(x)$).

Suppose $f(x) = x^4 - 14x^2 + 9 = (x^2+bx+c)(x^2+dx+e)$.

Then $x^4 - 14x^2 + 9 = x^4 + (d+b)x^3 + (c+e+bd)x^2 + (cd+eb)x + ec$, so

$$\begin{cases} d+b=0 & \to d=-b \\ c+e+bd=-14 & \\ cd+eb=0 & \to eb=cb \to e=c \text{ or } b=0 \\ ec=9 & \end{cases}$$

If $e=c$, then $e=c=3$ or $e=c=-3$, so $6-b^2=-14 \Rightarrow b^2=20$ or $-6-b^2=-14 \Rightarrow b^2=8$. In either case there is no $b \in \mathbb{Z}$ satisfying this.

If $b=0$, then $c+e=-14$ & $ec=9$. Testing all combinations of $e, c \in \mathbb{Z}$ s.t. $ec=9$, we see that none satisfy $c+e=-14$.

Therefore $f(x)$ is irreducible over $\mathbb{Q}$.

Thus $|\mathbb{Q}(\sqrt{2}+\sqrt{5}):\mathbb{Q}| = 4$, while $|\mathbb{Q}(\sqrt{2}):\mathbb{Q}| = 2$, so $\sqrt{2}+\sqrt{5} \notin \mathbb{Q}(\sqrt{2})$.

Since $\sqrt{2} \in \mathbb{Q}(\sqrt{2})$, it follows that $\sqrt{5} \notin \mathbb{Q}(\sqrt{2})$, & since $\sqrt{10} = \sqrt{5}\cdot\sqrt{2}$, it also follows that $\sqrt{10} \notin \mathbb{Q}(\sqrt{2})$. [To see this, suppose $\sqrt{10} \in \mathbb{Q}(\sqrt{2})$. Then $\sqrt{5} = \sqrt{2}^{-1}\sqrt{10} \in \mathbb{Q}(\sqrt{2})$, which is a contradiction.]

$\operatorname{irred}(\sqrt{10}, \mathbb{Q}) = x^2 - 10$, & $x^2 - 10 \in \mathbb{Q}(\sqrt{2})[x]$, so $|\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}(\sqrt{2})| = 2$.

Since $\sqrt{2}+\sqrt{5} = \sqrt{2} + \sqrt{2}^{-1}\sqrt{10} \in \mathbb{Q}(\sqrt{2},\sqrt{10})$, we know $\mathbb{Q}(\sqrt{2}+\sqrt{5}) \leq \mathbb{Q}(\sqrt{2},\sqrt{10})$.

Thus we have the following diagram:

$$\begin{tikzcd}
 & \mathbb{Q}(\sqrt{2},\sqrt{10}) \arrow[dl, dash, "2"'] \arrow[d, dash] \\
\mathbb{Q}(\sqrt{2}) \arrow[dr, dash, "2"'] & \mathbb{Q}(\sqrt{2}+\sqrt{5}) \arrow[d, dash, "4"] \\
 & \mathbb{Q}
\end{tikzcd}$$

From the diagram we see that:

$$|\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}| = |\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}(\sqrt{2})|\cdot|\mathbb{Q}(\sqrt{2}),\mathbb{Q}|$$
$$= 2\cdot 2 = 4.$$

Also from the diagram we see that:

$$|\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}| = |\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}(\sqrt{2}+\sqrt{5})|\cdot|\mathbb{Q}(\sqrt{2}+\sqrt{5}):\mathbb{Q}|$$

Thus, $4 = |\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}(\sqrt{2}+\sqrt{5})|\cdot 4$

$\Rightarrow |\mathbb{Q}(\sqrt{2},\sqrt{10}):\mathbb{Q}(\sqrt{2}+\sqrt{5})| = 1$.

Therefore, $\mathbb{Q}(\sqrt{2},\sqrt{10}) = \mathbb{Q}(\sqrt{2}+\sqrt{5})$.