Feldman Solutions

(2) (a) $z = x^2 - 2xy - y^2 - 8x + 4y = f(x,y)$

$f_x(x,y) = 2x - 2y - 8 = 0 \rightsquigarrow x - y - 4 = 0 \rightsquigarrow x = y + 4$

$f_y(x,y) = -2x - 2y + 4 = 0 \rightsquigarrow x + y - 2 = 0 \rightsquigarrow y + 4 + y - 2 = 0$

$2y + 2 = 0$

$y = -1$

$x = -1 + 4 = 3$

The tangent plane is horizontal at $\boxed{(3,-1)}$

(b) $x^3 z - \sin(x^2+y^2+z^2) - y^3 = 0.$

$$\frac{\partial}{\partial y}\left(x^3 z - \sin(x^2+y^2+z^2) - y^3\right) = \frac{\partial}{\partial y}(0)$$

$$x^3 \cdot \frac{\partial z}{\partial y} - \cos(x^2+y^2+z^2)\left(2y + 2z \cdot \frac{\partial z}{\partial y}\right) - 3y^2 = 0$$

$$\frac{\partial z}{\partial y}\left(x^3 - 2z\cos(x^2+y^2+z^2)\right) = 3y^2 + 2y\cos(x^2+y^2+z^2)$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{3y^2 + 2y\cos(x^2+y^2+z^2)}{x^3 - 2z\cos(x^2+y^2+z^2)}}$$

(4) $x^2+y^2 \le 1$, $T = 2x^2 + y^2 - y$.

Interior: $T_x = 4x = 0 \Rightarrow x = 0$

$T_y = 2y - 1 \Rightarrow y = 1/2$

Boundary: $g(x,y) = x^2 + y^2$

$\vec{\nabla} T = \langle 4x, 2y-1 \rangle$, $\vec{\nabla} g = \langle 2x, 2y \rangle$

$$\begin{cases} 4x = \lambda 2x \longrightarrow \lambda = 2 \text{ or } x = 0,\ y = \pm 1 \\ 2y - 1 = \lambda 2y \\ x^2 + y^2 = 1 \end{cases}$$

$\lambda = 2$: $2y - 1 = 4y$

$-1 = 2y$

$y = -1/2$

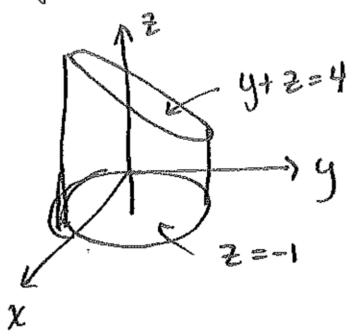
$x^2 = 3/4$

$x = \pm\sqrt{3}/2$

(x,y)	T
$(0, 1/2)$	$1/4 - 1/2 = -1/4$
$(\pm\frac{\sqrt{3}}{2}, -\frac{1}{2})$	$2 \cdot \frac{3}{4} + \frac{1}{4} + \frac{1}{2} = 9/4$
$(0,1)$	$1 - 1 = 0$
$(0,-1)$	$1 + 1 = 2$

$\boxed{\text{Max at } (\pm\frac{\sqrt{3}}{2}, -\frac{1}{2}) \quad \text{Min at } (0, 1/2)}$

⑤ $x^2+y^2=4,\ z=-1,\ y+z=4$



$$\int_0^{2\pi}\int_0^2\int_{-1}^{4-r\sin\theta} r\,dz\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^2 r(4-r\sin\theta+1)\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^2 5r-r^2\sin\theta\,dr\,d\theta$$

$$=\int_0^{2\pi} \frac{5r^2}{2}-\frac{r^3}{3}\sin\theta\,\Big|_0^2\,d\theta$$

$$=\int_0^{2\pi} 10-\frac{8}{3}\sin\theta\,d\theta$$

$$=10\theta+\frac{8}{3}\cos\theta\,\Big|_0^{2\pi}$$

$$=20\pi+\frac{8}{3}-\frac{8}{3}=\boxed{20\pi}$$

⑥ $\vec{F}=\langle \sin y,\ x\cos y+\cos z,\ 2z-y\sin z\rangle$

(a) $\int \sin y\,dx = x\sin y + C(y,z)$

$\int x\cos y+\cos z\,dy = x\sin y + y\cos z + D(x,z)$

$\int 2z-y\sin z\,dz = z^2+y\cos z+E(x,y)$

Let $f(x,y,z)=x\sin y+y\cos z+z^2$. Then $\vec{F}=\vec{\nabla}f$, so $\vec{F}$ is conservative, with potential fcn $f(x,y,z)=x\sin y+y\cos z+z^2$.

(b) $\vec{r}(0)=\langle 0,0,0\rangle$, $\vec{r}(\frac{\pi}{2})=\langle 1,\pi/2,\pi\rangle$. Since $\vec{F}$ is conservative,

$$\int_C \vec{F}\cdot d\vec{r}=f(1,\pi/2,\pi)-f(0,0,0)=1\cdot 1+\frac{\pi}{2}\cdot(-1)+\pi^2-0-0-0$$

$$=\boxed{1-\frac{\pi}{2}+\pi^2}$$

(7) $f(x,y,z) = 3x - y - 3z$, $\quad x + y - z = 0 \;\to\; z = x+y$, $\quad x^2 + 2z^2 = 1$

Use the first constraint to eliminate z:

$h(x,y) = f(x,y,x+y) = 3x - y - 3(x+y) = -4y$

$g(x,y) = x^2 + 2(x+y)^2 = 3x^2 + 4xy + 2y^2 = 1$

Now use Lagrange multipliers:

$\vec{\nabla} h = \langle 0, -4 \rangle$

$\vec{\nabla} g = \langle 6x+4y,\ 4x+4y \rangle$

$$\begin{cases} 0 = \lambda(6x+4y) \\ -4 = \lambda(4x+4y) \\ 3x^2 + 4xy + 2y^2 = 1 \end{cases}$$

Note: from 2nd eqn, $\lambda \neq 0$, as $-4 \neq 0(4x+4y)$

so $6x + 4y = 0$

$3x = -2y$

$x = -\frac{2}{3}y$

$3\left(-\frac{2}{3}y\right)^2 + 4\left(-\frac{2}{3}y\right)y + 2y^2 = 1$

$\frac{4}{3}y^2 - \frac{8}{3}y^2 + 2y^2 = 1$

$\frac{2}{3}y^2 = 1$

$y^2 = \frac{3}{2}$

$y = \pm\sqrt{\frac{3}{2}} \to x = -\frac{2}{3}y$

(x,y)	$h(x,y)$
$\left(-\frac{2}{3}\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$	$-4\sqrt{3/2}$
$\left(\frac{2}{3}\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}\right)$	$4\sqrt{3/2}$

$$\boxed{\text{So } f \text{ has a min at } \left(-\tfrac{2}{3}\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}, \underbrace{\sqrt{\tfrac{3}{2}} - \tfrac{2}{3}\sqrt{\tfrac{3}{2}}}_{z=x+y}\right) \text{ and a max at } \left(\tfrac{2}{3}\sqrt{\tfrac{3}{2}}, -\sqrt{\tfrac{3}{2}}, \tfrac{2}{3}\sqrt{\tfrac{3}{2}} - \sqrt{\tfrac{3}{2}}\right)}$$