1. We saw on the last worksheet how to use a line integral to compute the work done by a given vector field. In this exercise, we compute a different quantity, known as **flux**. The integral we'll be computing is

$$\int_C \vec{F} \cdot \vec{N}\, ds = \int_{t=a}^{t=b} \vec{F} \cdot \vec{N} \|\vec{r}\,'\|\, dt$$

where $\vec{F}$ is the vector field we're integrating, $\vec{N}$ is a unit normal vector field for the curve C, and $\vec{r}$ is a parametrization of C. The convention (in this class, not always!) is that $\vec{N}$ should point outwards from C.

The physical interpretation of flux is as follows: if $\vec{F}$ describes the velocity of some fluid in the xy plane, then the flux is how much fluid is crossing the curve C.

   (a) Suppose C is the unit circle centered at the origin, oriented counter-clockwise. Find a parametrization $\vec{r}(t)$ of C.

   (b) Find the unit normal vector field $\vec{N}(t)$ for the curve C. (Remember finding $\vec{T}$, $\vec{N}$, and $\vec{B}$?) Does N point outwards from C? If it points inwards, your answer will be negated.

   (c) Suppose $\vec{F} = \langle x, y \rangle$. Compute the flux of F across C.

   (d) Suppose instead that $\vec{F} = \langle 1, 0 \rangle$. Without computing anything, and only using the physical interpretation, what is the flux of $\vec{F}$ over C? What is the flux of $\vec{F}$ over any closed curve (a loop)?

   (e) Again with $\vec{F} = \langle 1, 0 \rangle$, what is the flux if C is a horizontal line? How about a vertical line?

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2. Let C be the ellipse $16x^2 + y^2 = 16$, oriented counter-clockwise, and let $\vec{F}(x, y) = \langle x, y \rangle$. Find the work done by $\vec{F}$ on a particle moving around C. Find the flux of $\vec{F}$ across C.

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3. Let C be the closed path that consists of the upper semicircle $x^2 + y^2 = 9$ from the point $(3, 0)$ to the point $(-3, 0)$, followed by the straight line segment from $(-3, 0)$ to $(3, 0)$. Let $\vec{F}(x, y) = \langle x^2, y^2 \rangle$. Find the work and flux, as in the previous problem.

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4. Find the flux of $\vec{F}$ across the triangle with vertices $(1, 0)$, $(0, 1)$, and $(-1, 0)$.

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5. Compute the following integrals

   (a) $\int_C x^2 + y^2\, ds$ where C is the line segment from $(0, 0)$ to $(1, 1)$.

   (b) $\int_C \vec{F} \cdot\, d\vec{r}$ where $\vec{F} = \langle x, y \rangle$ and C is as above.

   (c) $\int_C \vec{F} \cdot \vec{N}\, ds$ where $\vec{F} = \langle x, y \rangle$ and C is as above.

6. Using what you know about flux, how might you determine if a closed curve traverses around the origin?