1. A vector field $\vec{F}$ is called **conservative** if there is some function f such that $\vec{\nabla} f = \vec{F}$. Such a function f is called a **potential function**. Line integrals of conservative vector fields can be computed by the following elegant formula.

   Suppose C is any (smooth) path from a point A to a point B. Then
   $$\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A).$$
   In particular, observe that the value of the integral doesn't depend on how we get from A to B. This means we can skip finding a parametrization for C, which is often the most difficult part.

   (a) Integrate $\vec{F} = x^2\vec{i} - y\vec{j}$ where C is any (smooth) path from $(0,0)$ to $(1,1)$. What if C is from $(0,0)$ to (a,b)?

   (b) Using your work from problem 2 on Worksheet 19, is $\vec{F} = y\vec{i} - x\vec{j}$ a conservative vector field?

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2. Consider the field
   $$\vec{F}(x,y) = \begin{pmatrix} 1+y \\ x \end{pmatrix}.$$
   Compute the work done by $\vec{F}$ along each of the following curves:

   (a) The straight line segment from $(-1,-1)$ to $(1,1)$.

   (b) The part of the curve $y = x^3$ from $(-1,-1)$ to $(1,1)$.

   (c) The unit circle, oriented counterclockwise. (*Hint: use* $\cos^2(t) - \sin^2(t) = \cos(2t)$.)

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3. In the previous problem, is there anything interesting about the answers you got? Was this just a coincidence, or is there a deeper reason why this happened?

   (a) What do the curves in (a) and (b) above have in common?

   (b) What is special about the curve in (c)?

   (c) What can you say about $\vec{F}$? Prove it.

   (d) What is the work done by $\vec{F}$ along the connected path consisting of the straight line segment from $(-1,-1)$ to $(1,-1)$ followed by the straight line segment from $(1,-1)$ to $(1,1)$?

   (e) Find a potential function for $\vec{F}$.

   (f) Check that we get the answer in (d) by using the potential function found in (e).

   (g) Find the work done by $\vec{F}$ along **any** curve that starts at $(1,3)$ and ends at $(2,5)$.

   (h) Find the work done by $\vec{F}$ along the curve
   $$\vec{x}(t) = \begin{pmatrix} e^t\cos(2\pi t) \\ e^t\sin(2\pi t) \end{pmatrix}, \qquad 0 \le t \le 1.$$
