

1. A particle moves around the unit circle in the xy -plane. Its position at time t is $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$, where x and y are differentiable functions of t . Suppose $\vec{v}(t) \cdot \vec{i} = y(t)$, where $\vec{v}(t)$ is the velocity vector. Can you tell if the motion clockwise or counterclockwise? (Hint: Try to solve for dy/dt and see what you get...)
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2. Given the velocity functions below, find the particle's position subject to the given initial conditions.

(a) $\vec{v}(t) = (180t)\vec{i} + (180t - 16t^2)\vec{j}$ with $\vec{r}(0) = 100\vec{j}$.

(b) $\vec{v}(t) = (t^3 + 4t)\vec{i} + t\vec{j} + 2t^2\vec{k}$ with $\vec{r}(0) = \vec{i} + \vec{j}$.

3. Show that if a curve has constant speed, then its acceleration is always perpendicular to its velocity. (Hint: Rewrite the condition "constant speed" in terms of the dot product, and differentiate.)
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4. Show that if a particle always moves along a circle, then its velocity vector is always tangent to the circle. (Hint: Rewrite the condition "moves along a circle" in terms of the dot product. What does it mean to be tangent to a circle?)
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5. Suppose $\vec{x}(t) = \langle R \cos(\omega t), R \sin(\omega t) \rangle$.

(a) Find $\|\vec{v}(t)\|$.

(b) Write $\vec{a}(t)$ in terms of $\vec{x}(t)$.

(c) Rewrite Newton's formula ($\vec{F} = m\vec{a}$) in terms of $\vec{x}(t)$, and find $\|\vec{F}\|$ in terms of $\vec{v}(t)$. You should end up with the familiar formula for centrifugal force.
