

1. Determine whether each of the following vector fields is conservative, and then compute the work integral of \vec{F} along the given parametrized curve. You may use any rules/theorems you know.

(a) $\vec{F} = \langle 2y + 2x, 2x - 2y \rangle$ and $\vec{x}(t) = \langle t, t^2 \rangle$ for $0 \leq t \leq 1$

(b) $\vec{F} = \langle x - y, x + y \rangle$ and $\vec{x}(t) = \langle t, 2t \rangle$ for $0 \leq t \leq 1$

(c) $\vec{F} = \langle 2x, 2y \rangle$ and $\vec{x}(t) = \langle \sin(\pi t), e^{t^2-t} \rangle$ for $0 \leq t \leq 1$

2. For each integral, first change the order of integration, and then compute the integral.

(a)

$$\int_0^2 \int_{\sqrt{x}}^2 y \, dy \, dx$$

(b)

$$\int_0^1 \int_{-y}^y x^2 \, dx \, dy$$

3. If D is the cylinder of radius 2 and height 3 defined by $x^2 + y^2 \leq 4$ and $0 \leq z \leq 2$, compute the following integral

$$\iiint_D (x^2 + y^2 - z^2) dx dy dz$$

4. Let $f(x, y) = e^{x-2y}$.

(a) Use implicit differentiation to calculate $\frac{dy}{dx}$ at $(x_0, y_0) = (2, 1)$.

(b) Find a formula for the tangent line to the level set $z = 1$ at the point $(x_0, y_0) = (2, 1)$.

5. Let $f(x, y) = 2xy - x^2 - y$ and D be the domain defined by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find the minimum and maximum values of f on the closed domain.