

Problems from Section, Thursday 9/10

1.

$$\begin{aligned}\int \sin^6 x \cos^3 x \, dx &= \int \sin^6 x \cos^2 x \cos x \, dx \\ &= \int \sin^6 x (1 - \sin^2 x) \cos x \, dx \quad u = \sin x, du = \cos x \, dx \\ &= \int u^6 (1 - u^2) \, du \\ &= \int (u^6 - u^8) \, du \\ &= \frac{1}{7} u^7 - \frac{1}{9} u^9 + C \\ &= \frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C\end{aligned}$$

2.

$$\begin{aligned}\int \sin(1/2x) \cos(1/2x) \, dx &= \frac{1}{2} \int \sin x \, dx \\ &= -\frac{1}{2} \cos x + C\end{aligned}$$

or

$$\begin{aligned}\int \sin(1/2x) \cos(1/2x) \, dx &= 2 \int u \, du, \quad u = \sin(1/2x), du = \frac{1}{2} \cos(1/2x) \, dx \\ &= 2 \sin^2(1/2x) + C\end{aligned}$$

3.

$$\begin{aligned}\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx, \quad u = \cos x, du = -\sin x \, dx \\ &= -\int \frac{1}{u} \, du \\ &= -\ln \|u\| + C \\ &= -\ln |\cos x| + C\end{aligned}$$

4.

$$\begin{aligned}\int \sin^4 x \, dx &= \int (\sin^2 x)^2 \, dx \\ &= \int \left(\frac{1 - \cos(2x)}{2} \right)^2 \, dx \\ &= \frac{1}{4} \int (1 - 2\cos(2x) + \cos^2(2x)) \, dx \\ &= \frac{1}{4} \int (1 - 2\cos(2x) + \frac{1}{2}(1 + \cos(4x))) \, dx \\ &= \frac{1}{4} \left(x - \sin(2x) + \frac{1}{2} \left(x + \frac{1}{4} \sin(4x) \right) \right) + C \\ &= \frac{1}{32} (12x - 8\sin(2x) + \sin(4x))\end{aligned}$$