Exercises 14: 5, 6, 7, 13, 14, 30, 31, 34, 38*, 39

Exercises 15: 3, 4, 6, 11, 35, 36

Additional exercises:

1. Recall that $SL_n(\mathbb{R}) = \{A \in GL_n(\mathbb{R}) \mid \det(A) = 1\}$. Prove that $SL_n(\mathbb{R})$ is a subgroup of $GL_n(\mathbb{R})$. Then prove that it is a normal subgroup in two different ways:
   (a) first, using the fact that H is a normal subgroup of G if and only if $gHg^{-1} \subset H$ for all $g \in G$.
   (b) second, by finding a map from $GL_n(\mathbb{R})$ to a group such that $SL_n(\mathbb{R})$ is the kernel of the map. (Hint: Use the definition of $SL_n(\mathbb{R})$ to help you find this map.)
2. Prove that any quotient of a cyclic group is cyclic.

Challenge problem (optional, not to be turned in):

1. Prove that, up to isomorphism, there are exactly two groups of order 6, using the following steps.
   (a) Let G be a group of order 6. Show that G must have an element of order 2. (You actually already proved this on a previous assignment; see if you can remember why this was true!) Show that it cannot be true that every non-identity element of G can have order 2. (Hint: Show that if every non-identity element had order 2 then it would be possible to construct a subgroup of order 4.)
   (b) Show that if G contains elements $a, b \in G$ such that $|a| = 3$ and $|b| = 2$, then either G is cyclic or $ab \neq ba$.
   (c) Show that if G is not cyclic, then we can list the elements of G as $\{e, a, a^2, b, ab, a^2b\}$. What element on the list must be ba?
   (d) If G is not cyclic, draw the group table for G and and the group table for S_3.
   (e) Conclude that G is isomorphic to either $\mathbb{Z}_6$ or S_3.

*Recall that the map $i_g \colon G \to G$ is called an *inner automorphism* and is defined by $i_g(x) = gxg^{-1}$ for all $x \in G$.