Exercises 5: 14, 28, 29, 46, 47, 51, 52, 53, 57

Exercises 6: 33-37, 49, 50, 55

Additional exercises:

1. In Exercises 5 #57 you showed that a group with no proper nontrivial subgroups is cyclic. What can you say about the order of such a group?

2. Let $G = GL(3, \mathbb{R})$, the group of 3×3 invertible matrices under multiplication.

   (a) Let L be the set of lower triangular real 3×3 matrices with ones on the diagonal, i.e.,
   $$L = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$
   Prove L is a subgroup of G.

   (b) Note that G is not abelian, but it does have abelian subgroups, such as the trivial subgroup $\{I_3\}$, where I_3 is the 3×3 identity matrix. Find two nontrivial abelian subgroups H and K of G which are not isomorphic to each other. Justify all parts (subgroup, abelian, not isomorphic).

3. Draw the subgroup diagram for the dihedral group D_4. (Note: D_4 has 10 subgroups.)

4. (a) Prove that the group of positive rational numbers under multiplication, $(\mathbb{Q}^+, \cdot)$, is generated by the (infinite) set $\{\frac{1}{p} \mid p \text{ is prime}\}$.

   (b) Prove that $(\mathbb{Q}^+, \cdot)$ cannot be generated by any *finite* set of elements.