Exercises 5: 14, 20, 28, 46, 47, 51, 52

Exercises 6: 33-37, 44, 49, 50, 55

Additional exercises:

1. Let $G = GL(3, \mathbb{R})$, the group of 3×3 invertible matrices under multiplication.

   (a) Let L be the set of lower triangular real 3×3 matrices with ones on the diagonal, i.e.,
   $$L = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$
   Prove L is a subgroup of G.

   (b) Note that G is not abelian, but it does have abelian subgroups, such as the trivial subgroup $\{I_3\}$. Find two nontrivial abelian subgroups H and K of G which are not isomorphic to each other. Justify all parts (subgroup, abelian, not isomorphic).

2. Let $\phi : G \to H$ be a group homomorphism.

   (a) Prove that $\phi(e_G) = e_H$, where e_G is the identity element of G and e_H is the identity element of H.

   (b) Prove that if a and b are inverses of each other in G, the $\phi(a)$ and $\phi(b)$ are inverses of each other in H. Conclude that if an element x is its own inverse in G, then $\phi(x)$ is its own inverse in H.

   (c) If G is abelian, prove that $\phi(g_2)\phi(g_1) = \phi(g_1)\phi(g_2)$ for all $g_1, g_2 \in G$. Does this prove that H is also abelian? Why or why not?