Exercises 27: 2, 4, 5, 19, 24, 30

Exercises 29: 7, 10, 12, 29, 30, 31, 35, 36

Additional exercises:

1. The goal of this problem is to prove that if R and S are rings with unity and K is an ideal of $R \times S$, then there are ideals $I \leq R$ and $J \leq S$ such that $K = I \times J$.

   (a) Define $I = \{r \in R \mid \text{ there exists } s \in S \text{ such that } (r, s) \in K\}$ and $J = \{s \in S \mid \text{ there exists } r \in R \text{ such that } (r, s) \in K\}$. Prove that I and J are ideals of R and S, respectively.

   (b) Prove that $K \subseteq I \times J$.

   (c) Prove that $I \times J \subseteq K$. For this part, we want to show that if $r \in I$ and $s \in J$, then $(r, s) \in K$. Consider the elements $s' \in S$ and $r' \in R$ such that $(r, s'), (r', s) \in K$. (Why do such elements exist?) Since K is an ideal, $(a, b)(r, s') \in K$ and $(a, b)(r', s) \in K$ for all $(a, b) \in R \times S$. Find an appropriate choice (or choices) of (a, b) so that you can conclude that $(r, s) \in K$.