1. Consider the parametric curve given by:

$$\mathbf{x}(t) = \begin{pmatrix} e^t \cos t \\ e^t \sin t \\ e^t \end{pmatrix}$$

   (a) Find the unit tangent, normal, and binormal vectors.

   (b) Find the curvature κ.

   (c) Find the arc length from $t = 0$ to $t = \pi$.

   (d) Parametrize the line that is tangent to $\mathbf{x}(t)$ at the point t_0.

   (e) Find the point where the above tangent line intersects the xy-plane. Let $\mathbf{y}(t_0)$ be the position vector for this point. As t_0 varies, sketch the curve that $\mathbf{y}(t_0)$ traces.

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2. Consider the cycloid $\mathbf{x}(t) = \begin{pmatrix} t - \sin t \\ 1 - \cos t \end{pmatrix}$. Find the arc length of one period, $0 \leq t \leq 2\pi$.

   Hint: *you can use the following trig identity.*

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

   Also, be careful with absolute values.

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3. Consider the curve $\mathbf{x}(t) = \begin{pmatrix} a\cos(t) \\ a\sin(t) \\ bt \end{pmatrix}$.

   (a) Determine what kind of curve this is (circle, helix, cycloid, etc. – be precise!).

   (b) Find the unit tangent, normal, and binormal vectors for $\mathbf{x}$.

   (c) Find the curvature vector scalar curvature $\kappa(t)$.