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1. (a) Let $\mathbf{u} \neq \mathbf{0}$. Describe the vectors $\mathbf{v}$ such that $\mathbf{u} \cdot \mathbf{v} \leq 0$. Do the same for the vectors such that $\mathbf{u} \cdot \mathbf{v} \geq 0$. Your description should be geometric.

   (b) If $\|\mathbf{u}\| = a$ and $\|\mathbf{v}\| = b$, what are the maximum and minimum values for $\mathbf{u} \cdot \mathbf{v}$?

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2. (a) Find all unit vectors in the xy-plane that are perpendicular to $3\mathbf{i} - 4\mathbf{j}$.

   (b) Find all values of c so that $\mathbf{i} + c\mathbf{j}$ makes an angle of $\frac{\pi}{4}$ with $2\mathbf{i} - \mathbf{j}$.

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3. Consider the line in the xy-plane with equation $y = \frac{5}{12}x + 11$. Find all unit vectors parallel to this line and all unit vectors perpendicular to this line.

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4. Suppose $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$. Can we conclude that $\mathbf{v} = \mathbf{w}$? Explain or give a counterexample. (Hint: $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ can be written as $\mathbf{u} \cdot (\mathbf{v} - \mathbf{w}) = 0$. Would that mean $\mathbf{v} - \mathbf{w} = \mathbf{0}$?)

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5. For the time being, use the following definition of the dot product.

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \, \|\mathbf{w}\| \cos \theta.$$

   (a) Using this definition, find $\mathbf{i} \cdot \mathbf{j}$, $\mathbf{i} \cdot \mathbf{k}$, $\mathbf{j} \cdot \mathbf{k}$, $\mathbf{i} \cdot \mathbf{i}$, $\mathbf{j} \cdot \mathbf{j}$, and $\mathbf{k} \cdot \mathbf{k}$.

   (b) Assuming that the commutative, associative, and distributive properties of the dot product hold (see page 10 of your text if you're unfamiliar with these properties), deduce that

$$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

   where $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ and $\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$.

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6. Now we change our definition of the dot product. This time, define the dot product as

$$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

   with $\mathbf{v}$ and $\mathbf{w}$ as above. From this expression, you could deduce the algebraic properties found on page 10 fairly easily. You don't need to do this right now, and can take these properties for granted if you wish.

(a) Draw the vectors $\mathbf{v}$ and $\mathbf{w}$ so that their tail ends (the "starting points") are at the same point. Label the angle between $\mathbf{v}$ and $\mathbf{w}$ as θ in your picture. Where is the vector $\mathbf{v}-\mathbf{w}$ in your picture? (*Hint: you get a triangle!*)

(b) Using the law of cosines, write an equation expressing the relationship between $\cos\theta$ and the magnitudes of the vectors $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v}-\mathbf{w}$.

(c) In this last equation, rewrite $\|\mathbf{v}-\mathbf{w}\|^2$, $\|\mathbf{v}\|^2$, $\|\mathbf{w}\|^2$ in terms of dot products, and simplify using the algebraic properties. You should arrive at the definition we used in the previous exercise.

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7. In light of the previous two exercises, does it matter which of the two definitions of the dot product we use? Give a brief explanation of when you might want to use one definition over the other.

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8. (a) Let $\mathbf{v}$ and $\mathbf{w}$ be fixed. When will $\mathbf{v}+\mathbf{w}$ and $\mathbf{v}-\mathbf{w}$ be perpendicular?

   (b) Draw one diagram that shows $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v}+\mathbf{w}$, and $\mathbf{v}-\mathbf{w}$.

   (c) Suppose we have a parallelogram whose diagonals are of equal length. What can we conclude about the parallelogram?

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9. Using general properties of vectors, show that

$$\|\mathbf{v}+\mathbf{w}\|^2+\|\mathbf{v}-\mathbf{w}\|^2=2\left(\|\mathbf{v}\|^2+\|\mathbf{w}\|^2\right).$$

Hint: write lengths in terms of dot products and simplify!

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10. Using general properties of vectors, show that

$$(\mathbf{v}\times\mathbf{w})\cdot(\mathbf{v}\times\mathbf{w})=(\mathbf{v}\cdot\mathbf{v})(\mathbf{w}\cdot\mathbf{w})-(\mathbf{v}\cdot\mathbf{w})^2.$$

Hint: first make sure the equation holds if at least one of the vectors is zero. Otherwise let θ be the angle between $\mathbf{v}$ and $\mathbf{w}$, write dot products in terms of lengths and θ, and simplify!

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11. Find the area of the triangle with vertices $A(3,1,-1)$, $B(2,0,1)$, $C(1,-2,0)$.

Hint: this triangle is half of some parallelogram. Can you find the area of that parallelogram?
