MATH 222(1,2,4) Fall 2015
Quiz 5RBSolutions

Please inform your TA if you find any errors in the quiz solutions.

1. (*4 points*)

1. (2 *points*) Suppose that

$$y' = y^2 + x, \qquad y(2) = -1$$

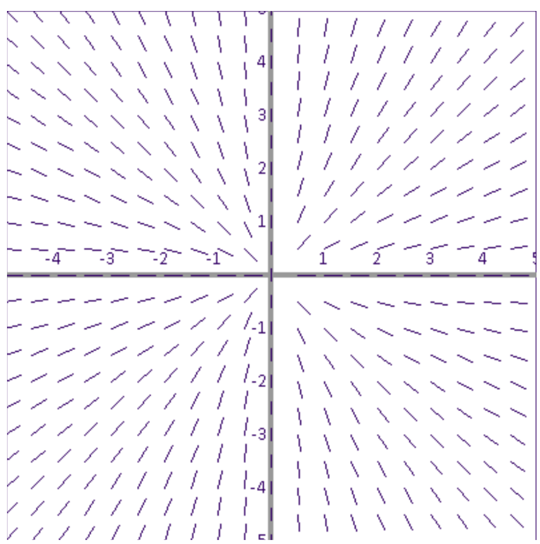
Use Euler's method with step size 0.1 to approximate $y(2.1)$.

Solution:

$$\begin{aligned} y(2) &= -1 \\ y(2.1) &\approx y(2) + 0.1 * y'(y) \\ &= -1 + 0.1 * ((-1)^2 + 1 * 2) \\ &= -0.7 \end{aligned}$$

2. (2 *points*) Circle the differential equation that corresponds to the slope field shown below.

$y' = y/x$ $\qquad$ $y' = \sin(x)$ $\qquad$ $y' = x + y$ $\qquad$ $y' = -x/y$



Solution: $y' = y/x$

2. (*6 points*) Find a solution to the initial value problem

$$\begin{gathered} x\frac{dy}{dx} + 2y = \frac{e^{2x}}{x} \\ y(1/2) = 0 \end{gathered}$$

Solution: We being by writing the differential equation in standard form as

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{e^{2x}}{x^2}$$

The integrating factor for this problem is $m(x) = e^{\int \frac{2}{x}dx} = e^{2\ln(x)} = x^2$. Multiplying through by x^2 converts this problem to

$$\begin{aligned} x^2\frac{dy}{dx} + 2xy &= e^{2x} \\ \frac{d(x^2y)}{dx} &= e^{2x} \\ \int d(x^2y) &= \int e^{2x}dx \\ x^2y &= \frac{1}{2}e^{2x} + C \\ y(x) &= \frac{e^{2x}}{2x^2} + \frac{C}{x^2} \end{aligned}$$

Susbtituting in the initial condition, we find that

$$0 = \frac{e}{1/2} + \frac{C}{1/4}$$

so that $y(x) = \frac{e^{2x}}{2x^2} + \frac{2e}{x^2}$.