

Name: _____

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MATH V1201, Section 12 - Midterm 2

November 18, 2015 (75 minutes)

This examination booklet contains 6 problems. There are 13 sheets of paper including the front cover and formula sheet at the back. This is a closed book exam. Do all of your work on the pages of this exam booklet. Show all your computations and justify/explain your answers (except for the true/false problems). Cross out anything you do not want graded. Calculators are NOT allowed.

You have 75 minutes to complete the exam. Do not begin until instructed to do so. When time is up, stop working and close your test booklet. Cell phones, headphones, laptops and other electronic devices are not allowed.

Problem	Possible score	Your score
1	16	
2	15	
3	21	
4	14	
5	24	
6	10(+10)	
Total	100(+10)	

Grades will be posted on CourseWorks.

1.

(a) (6 pts.) Find a vector-valued function $\vec{r}(t)$ that represents the curve of intersection of the surfaces

$$z = x^2 + 4y^2 \text{ and } y = x^2.$$

(b) (10 pts.) Let C be the smooth curve parameterized by the vector-valued function

$$\vec{p}(t) = \langle t, \sin(2t), \cos(2t) \rangle.$$

Find the \vec{T} - \vec{N} - \vec{B} frame at the point $(\pi, 0, 1)$ on the curve C .

2. Circle the word True or False to indicate your answer. (No explanation needed.)

(a) Suppose that $\vec{r}_1(s)$ and $\vec{r}_2(t)$ are two smooth parametrizations of the same *oriented* smooth curve C . (Recall that by using the word oriented, we mean that the two parametrizations move along C in the same direction as s and t increase.)

(i) (3 pts.) The curvature at a point P on C is the same whether it is calculated using \vec{r}_1 or \vec{r}_2 .

True False

(ii) (3 pts.) The \vec{T} - \vec{N} - \vec{B} -frame at a point P on C the same whether it is calculated using \vec{r}_1 or \vec{r}_2 .

True False

(b) (3 pts.) For any real numbers a, b , and c , the linearization $L(x, y)$ of the function $f(x, y) = ax + by + c$ at the point $(x_0, y_0) = (3, 2)$ is equal to $f(x, y)$.

True False

(c) (3 pts.) Let $\vec{a}(t)$, $\vec{b}(t)$, and $\vec{c}(t)$ be three vector-valued functions. Then

$$\frac{d}{dt}(\vec{a}(t) \cdot (\vec{b}(t) \times \vec{c}(t))) = \vec{a}'(t) \cdot (\vec{b}(t) \times \vec{c}(t)) + \vec{a}(t) \cdot (\vec{b}'(t) \times \vec{c}(t)) + \vec{a}(t) \cdot (\vec{b}(t) \times \vec{c}'(t)).$$

True False

(d) (3 pts.) Suppose that $f(x, y)$ is a continuous function on the xy -plane with the origin $(0, 0)$ removed. Also suppose that $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists and is equal to 0. Then it is possible that $\lim_{(x,y) \rightarrow (0,0)} f(x, y)^2$ does not exist.

True False

3. Evaluate the limit or show that it does not exist.

(a) (7 pts.) Find

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{x^2 + 2y^2}.$$

(b) (7 pts.) Find

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}.$$

(c) (7 pts.) Find

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2).$$

4. (14 pts.) Find the osculating circle of the parabola $x = y^2$ at the point $(0, 0)$.

5.

(a) (8 pts.) The volume of a cylinder of height h and radius r is given by $V = \pi r^2 h$. Find the rate at which the volume V is changing per second if the height is 3, the radius is 4, the height is increasing by $\frac{1}{2}$ a unit per second, and the radius is decreasing by $\frac{1}{2}$ a unit per second.

(b)

(i) (8 pts.) Suppose that the function $F(x, y)$ is defined by

$$F(x, y) = g(x^2 + y^2, y - x)$$

where $g(r, s) = rs$. Calculate the first partial derivatives of F with respect to x and y .

(ii) (5 pts.) What is the linearization of $F(x, y)$ at $(1, 1)$?

(iii) (3 pts.) Suppose that y is implicitly defined as a function of x by the equation

$$F(x, y) = 0.$$

In terms of F_x and F_y , what is $\frac{dy}{dx}$? (You do not need to substitute your answer from part (i); just leave your answer in terms of F_x and F_y .)

6.

(a) (10 pts.) Write down and evaluate the arc length integral from $\vec{r}(0)$ to $\vec{r}(t)$ of the curve

$$\vec{r}(t) = \left(\frac{1}{t^2 + 1} - \frac{1}{2} \right) \vec{i} + \frac{t}{t^2 + 1} \vec{j}.$$

Hint: The formula sheet at the end of the booklet may be helpful.

(b) (10 pts., extra credit) Reparametrize $\vec{r}(t)$ with respect to arc length and fully simplify. What is the curve?

Some useful formulas

Feel free to tear this sheet off from your exam booklet.

Arc length and curvature

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}, \vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}, \text{ and } \vec{B}(t) = \vec{T}(t) \times \vec{N}(t) \quad (1)$$

$$\kappa(t) = \left| \frac{d\vec{T}}{ds} \right| = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}, \kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}} \quad (2)$$

Motion in space

$$\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t) \quad (3)$$

$$\vec{v}(t) = v(t)\vec{T}(t) \quad (4)$$

$$\vec{a}(t) = v'(t)\vec{T}(t) + \kappa(t)v^2(t)\vec{N}(t) = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{|\vec{r}'(t)|}\vec{T}(t) + \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|}\vec{N}(t) \quad (5)$$

Trigonometry

$$\frac{d}{dx}(\tan x) = \sec^2 x, \frac{d}{dx}(\csc x) = -\csc x \cot x, \frac{d}{dx}(\sec x) = \sec x \tan x, \frac{d}{dx}(\cot x) = -\csc^2 x \quad (6)$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \quad (7)$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}, \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}, \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \quad (8)$$