PRACTICE MID TERM II
CALCULUS III

- The use of class notes, book, formulae sheet, calculator is not permitted.
- In order to get full credit, you **must** show all your work.
- Each solution must have a clearly labeled problem number and start at the top of a new page.
- You have **one hour and fifteen minutes**.
- Do not forget to write your name and UNI in the space provided below and on the notebook provided.

(1) Compute the length of the following parametric curve, for $0 \leq t \leq 2$.

$$\overrightarrow{\mathbf{r}}(t) = \left\langle 9t, (2t)^{\frac{3}{2}}, \frac{t^2}{2} \right\rangle$$

(2) Compute the tangential and normal components of the acceleration of a particle whose position at time t is given by $\overrightarrow{\mathbf{r}}(t) = \langle t, 2t, t^2 \rangle$. What is the curvature of its path at $t = 2$.

(3) Draw the contour map of $f(x, y) = \frac{y}{x^2+y^2}$.

(4) True/False. Justify your answer.

(a) $\frac{d}{dt}|\overrightarrow{\mathbf{r}}(t)| = |\overrightarrow{\mathbf{r}}'(t)|$.

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin(y)}{x^2 + y^2} = 0$$

(c) There exists a function $f(x, y)$ such that $f_x = x + y^2$ and $f_y = x - y^2$.

(d)

$$\lim_{(x,y)\to(0,0)} \sqrt{x^2 + y^2} \ln(x^2 + y^2) = 0$$

(5) Compute f_{xyz} for a function of three variables $f(x, y, z) = \sin^{-1}(xz) + x^2 e^{yz}$.

(6) Find the point(s) of intersection of the following parametric curves

$$\overrightarrow{\mathbf{r}}_1(t) = \langle \cos(t), \sin(t), t \rangle \qquad \overrightarrow{\mathbf{r}}_2(s) = \left\langle 1 + s, s, \frac{\pi}{2} s \right\rangle$$

(7) Assume that an object of mass 0.8 kg is thrown southwards with an initial speed of 30 m/s at an angle of 30 ° with the ground. If wind exerts a force of 4 N in the eastward direction, where does the object land?

(8) Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z^2 + x^2 y^2 + \cos(z) = 9$.