Midterm 1 Sample

Math UN1101: Calculus III, Section 2

Spring 2019

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Name: ____

Instructions:

- Print your name in the space above.
- Show your reasoning and intermediate computations.
- You have 75 minutes.
- No notes, books, calculators or any other electronic devices are allowed.
- Write answers in the space provided. If you need extra space, use the backs of pages.

1. Let $\vec{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\vec{b} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\vec{c} = \mathbf{i} + \mathbf{k}$. Compute the followings:

   (a) $\vec{a} + \vec{b} - \vec{c} =$

   Solution: $\vec{a} + \vec{b} - \vec{c} = -\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$

   (b) $\vec{a} \times \vec{c} =$

   Solution: $\vec{a} \times \vec{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{vmatrix} = 2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

   (c) Volume of the parallelpiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

   Solution:

   $$|\vec{b} \bullet (\vec{a} \times \vec{c})| = |-2| = 2$$

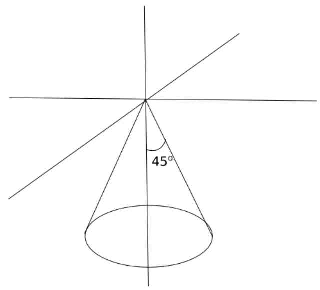
   (d) $\text{Proj}_{\vec{b}}\vec{c} =$

   Solution:

   $$\text{Proj}_{\vec{b}}\vec{c} = \frac{\vec{c} \bullet \vec{b}}{|\vec{b}|^2}\vec{b} = -\frac{1}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} + \frac{2}{9}\mathbf{k}$$

2. (a) Identify the surface in $\mathbb{R}^3$ described in spherical coordinates by $\phi = 3\pi/4$.

Solution: Downward cone with angle $\pi/4$.



(b) Find an equation for the surface in rectangular coordinates.

Solution:

$$\tan(\phi) = \frac{r}{z} = \frac{\sqrt{x^2+y^2}}{z}$$

Thus, $\sqrt{x^2+y^2} = -z$.

3. Let $\mathcal{P}_1$ be the plane $x + y + z = 1$, and $\mathcal{P}_2$ be the plane $x - y + z = 1$.

(a) Find cosine of the angle between $\mathcal{P}_1$ and $\mathcal{P}_2$.

Solution:

$$\vec{n}_1 = \langle 1, 1, 1 \rangle \quad \text{and} \quad \vec{n}_2 = \langle 1, -1, 1 \rangle$$

Thus, $\cos(\theta) = \frac{\vec{n}_1 \bullet \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{1}{3}$.

(b) Find symmetric equation for the line of intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$.

Solution:

Point on the line of intersection: $(0, 0, 1)$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2\mathbf{i} - 2\mathbf{k}$

Equation of the line: $x = -(z - 1)$ and $y = 0$

(c) The plane $\mathcal{P} : 2y - 2x - 2z = 3$ is parallel to either $\mathcal{P}_1$ or $\mathcal{P}_2$. Which one?

Solution: $\mathcal{P}_2$. (Explain your answer).

(d) Compute the distance between the parallel plane to $\mathcal{P}$, from part (c), and $\mathcal{P}$.

Solution: Point on $\mathcal{P}_2$: $p = (0, 0, 1)$

$$D = \frac{|2(0) - 2(0) - 2(1) - 3|}{|\sqrt{4 + 4 + 4}|} = \frac{5}{2\sqrt{3}}$$

4. Let L_1 and L_2 be the lines:

$$\begin{aligned} L_1: &\quad x = 1 + t, \quad y = 1 + 6t, \quad z = 2t \\ L_2: &\quad x = 1 + 2s, \quad y = 5 + 15s, \quad z = 6s - 2. \end{aligned}$$

(a) The line $x - 2 = -\frac{1-y}{6} = \frac{z-2}{2}$ is parallel to either L_1 or L_2. Which one?

Solution: L_1. (Explain your answer).

(b) The plane $2x + 12y + 4z = 5$ is orthogonal to either L_1 or L_2. Which one?

Solution:
L_1. (Explain your answer).

(c) Show that L_1 and L_2 are skew lines.

Solution: L_1 and L_2 are not parallel, because $\langle 1, 6, 2\rangle$ is not parallel to $\langle 2, 15, 6\rangle$.

$$\begin{cases} 1 + t = 1 + 2s \rightarrow t = 2s \\ 1 + 6t = 5 + 15s \\ 2t = 6s - 2 \rightarrow 4s = 6s - 2 \rightarrow 2s = 2 \rightarrow s = 1 \end{cases}$$

It follows from 1st and 3rd equations that $s = 1$ and $t = 2$. But these values don't satisfy in 2nd equation, so the lines don't intersect.

(d) Find an equation for a plane containing L_1 and which doesn't intersect L_2.

Solution:

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 2 \\ 2 & 15 & 6 \end{vmatrix} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

Point on L_1: $(1, 1, 0)$
Equation of plane: $6(x - 1) - 2(y - 1) + 3z = 0 \quad \rightarrow \quad 6x - 2y + 3z = 4.$

5. (a) Describe the trace of the quadric surface

$$z = x^2 + 2y^2$$

in the plane $z = 6$.

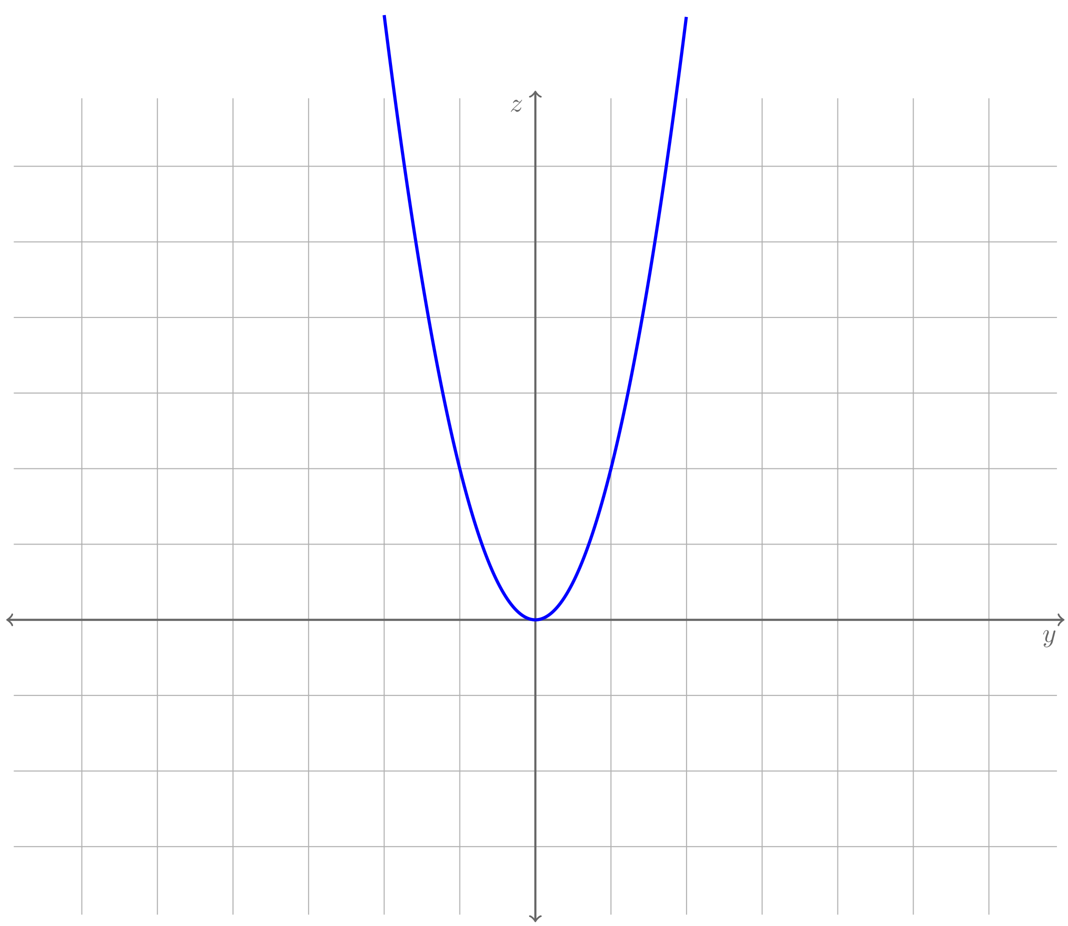
Solution: The $z = 6$ trace is an ellipse $1 = \frac{x^2}{6} + \frac{y^2}{3}$.

(b) Sketch the trace of the quadric surface

$$z = x^2 + 2y^2$$

in the yz-plane.

Solution: The $x = 0$ trace is the curve $z = 2y^2$, a parabola.



(c) Use the previous parts to sketch the quadric surface

$$z = x^2 + 2y^2.$$

Solution: The surface is an elliptic paraboloid which opens upward.