MID TERM I
CALCULUS III (26660 SEC. 2)

- The use of class notes, book, formulae sheet, calculator is not permitted.
- In order to get full credit, you **must** show all your work.
- You have **one hour and fifteen minutes**.
- Do not forget to write your name and UNI in the space provided below.

Print UNI ________________________

Print Name ________________________

For Grader's use only:

1	2	3	4	5	6	Total
___/15	___/15	___/20	___/15	___/20	___/15	___/100

Problem 1 (15 points) Consider the following two lines

$$L_1: \ x = 1 + t, \ y = 1 - t, \ z = 3t$$
$$L_2: \ x = 7 - 2s, \ y = 4 - s, \ z = 15 - 5s$$

(a) Find the point of intersection of L_1 and L_2.

(b) Find cosine of the *acute* angle between L_1 and L_2.

(c) Find the equation of the plane containing L_1 and L_2.

Problem 2 (15 points)

(a) Prove that there is no vector $\vec{v}$ such that

$$\langle 1,2,1\rangle \times \vec{v} = \langle 3,1,5\rangle$$

(b) Prove that the following three vectors are coplanar

$$\widehat{\imath} + 5\widehat{\jmath} - 2\widehat{k} \qquad 3\widehat{\imath} - \widehat{\jmath} \qquad 5\widehat{\imath} + 9\widehat{\jmath} - 4\widehat{k}$$

(c) For a non–zero vector $\vec{v}$ and a vector $\vec{u}$ prove that $\text{Proj}_{\vec{v}}(\vec{u}) \bullet \vec{v} = \vec{u} \bullet \vec{v}$.

Problem 3 (20 points) Consider the line $L : \vec{r} = \langle t, -t, 2t \rangle$ and the plane $\mathbb{P} : x+y+2z = 8$.

(a) Find the point of intersection of L and $\mathbb{P}$.

(b) If $\vec{v}$ is the direction vector of L and $\vec{n}$ is the normal vector of $\mathbb{P}$, compute $\mathrm{Orth}_{\vec{n}}(\vec{v})$. (recall $\mathrm{Orth}_{\vec{v}}(\vec{u}) = \vec{u} - \mathrm{Proj}_{\vec{v}}(\vec{u})$).

(c) Write the symmetric equations of the line passing through the point of part (a) parallel to the vector of part (b) (called the shadow of L in $\mathbb{P}$).

Problem 4 (15 points) True/False. You do not have to justify your answer. Please answer just T or F to each of the following.

(a) $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$.

(b) If $\vec{v} \bullet \vec{v} = 0$ then $\vec{v} = \vec{0}$.

(c) There is a unique line parallel to a given plane and passing through a given point.

(d) $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \bullet \vec{b}$.

(e) A pair of planes in $\mathbb{R}^3$ is either parallel or meets in a unique line.

Problem 5 (20 points)

(a) Let L_1 and L_2 be two parallel lines with direction vector $\vec{v}$ and passing through points P and Q respectively. Prove that the distance between L_1 and L_2 is given by:

$$\text{Distance between } L_1 \text{ and } L_2 \; = \frac{|\vec{PQ} \times \vec{v}|}{|\vec{v}|}$$

(b) Use the formula above to find the distance between

$$L_1 : \; x = 5 + 2t, \; y = 3 - 2t, \; z = 1 + t$$

$$L_2 : \frac{x - 3}{2} = -\frac{y}{2} = z + 1$$

Problem 6 (15 points) Consider the equation $x^2 + 4y^2 = a + z^2$.

(a) Sketch x and z traces of the surface, for $a = 1$.

(b) Use the traces of part (a) to sketch the surface (for $a = 1$).

(c) **Bonus: 5 points** Describe how the surface changes when $a \to 0^+$.