MATH UN1201 **Midterm Exam #1** FEBRUARY 19, 2020

Instructions:

- Complete all problems.
- You must show your work or justify your answer for all problems, except where otherwise stated. *Answers without work or justification will receive no credit (even if they are correct).*
- Put a BOX around your final answer.
- If you need more space, use the blank pages at the beginning and end of the exam. If you want me to grade work done any of those pages, clearly indicate this next to the appropriate problem.
- When writing vectors, you MUST use an arrow (in your work and in your answers). Failure to do so will result in loss of points.
- Leave your answers in an *exact* form (i.e., do not use decimal approximations). Simplify the arithmetic in your answers, but you do not have to simplify radicals or clear radicals from the denominators.

Problem 1		13
Problem 2		5
Problem 3		6
Problem 4		15
Problem 5		6
Problem 6		14
Total		59

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1. *(13 points)*

   (a) *(2 points each)* Let $\vec{v} = \langle 1, 0, 1 \rangle$ and $\vec{u} = \langle 1, 1, 0 \rangle$.

      i. Find the angle between $\vec{v}$ and $\vec{u}$.

      ii. Find the area of the parallelogram spanned by $\vec{v}$ and $\vec{u}$.

      iii. Find a unit vector orthogonal to $\vec{v}$ and $\vec{u}$.

(b) *(2 points)* Find a real number r such that vectors $\vec{a} = \langle 2, 3, 4 \rangle$ and $\vec{b} = \langle 3, r, -2 \rangle$ are orthogonal.

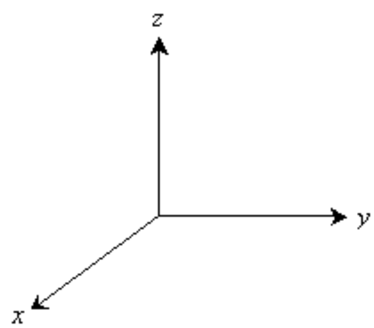
(c) *(3 points)* Are the points $A(1, 0, 1)$, $B(2, 3, 4)$, $C(-1, 0, 2)$, and $D(4, 6, -2)$ coplanar?

(d) *(2 points)* Find a vector that makes an angle of $\frac{\pi}{4}$ with $\vec{i}$.

2. The following equation is cylindrical coordinates.

$$\theta = \pi/4$$

(a) *(2 points)* Sketch the surface.



(b) *(3 points)* Write an equation of the surface in rectangular coordinates.

3. (a) *(3 points)* Let $\vec{u} = \langle -1, 0, 1 \rangle$ and $\vec{v} = \langle 4, 6, 9 \rangle$. Find $\operatorname{proj}_{\vec{u}} \vec{v}$.

(b) *(3 points)* The distance from a point P to the plane with normal vector $\vec{n}$ containing a point P_0 is given by the absolute value of

$$\frac{\vec{n} \cdot \overrightarrow{P_0P}}{|\vec{n}|}.$$

Use this formula to find the distance from the point $P(1, 2, 3)$ to the plane $3x + 4y - z = 5$.

4. *(5 points each)* Consider the point $P(1, 0, 1)$ and the line L given by

$$\vec{r} = \langle 2, -1, 3 \rangle + t\langle 3, 5, 1 \rangle.$$

(a) Find the equation of the plane $\mathbb{P}_1$ that is orthogonal to L and passes through P.

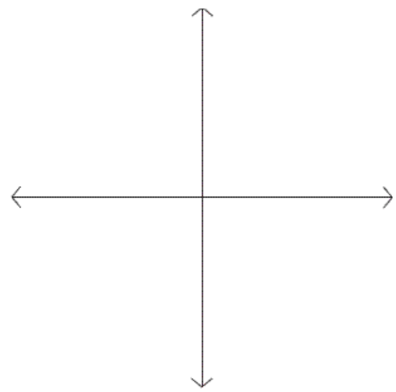
(b) Find the intersection of line L and the plane $\mathbb{P}_1$ from part (a).

(c) Find an equation for the line L_2 of intersection of the plane $\mathbb{P}_1$ from part (a) and the plane $\mathbb{P}_2$ with equation $3x + 2y + z = 7$.

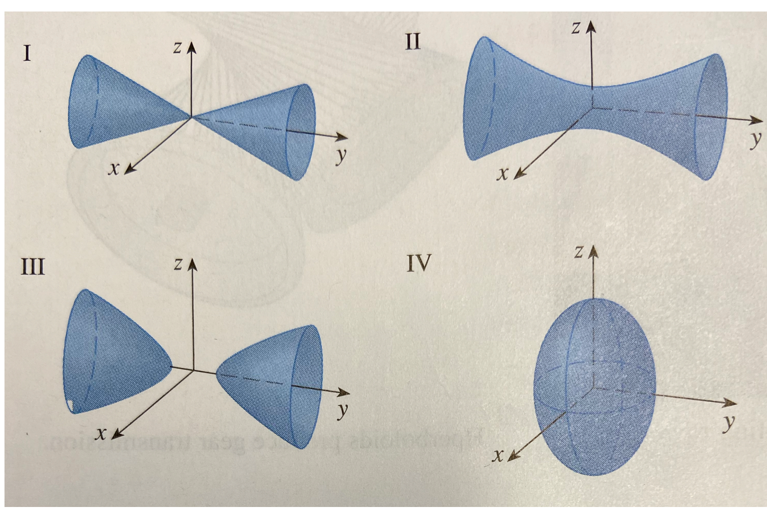
5. Consider the equation

$$\frac{x^2}{4} - y^2 + \frac{z^2}{9} = 1.$$

(a) *(4 points)* Sketch the traces $y = k$ for $k = -1, 0, 1$. Be sure to label the axes with the appropriate variables and label the curves with the corresponding value of k.



(b) *(2 points)* Which surface is the graph of the equation? Write your answer on the line provided.



Answer:____________

6. *(2 points each)* Determine if each statement is true or false, and write the (entire) word TRUE or FALSE in the space provided. You do not have to justify your answers, and no partial credit will be awarded.

   (a) The lines L_1 and L_2 are parallel, where

$$L_1\colon \frac{x-1}{2} = \frac{y+3}{4} = \frac{z}{2} \qquad \text{and} \qquad L_2\colon \begin{cases} x = 2 + \frac{1}{2}t \\ y = 1 + \frac{1}{4}t \\ z = \frac{1}{2}t. \end{cases}$$

   (b) In cylindrical coordinates $(2, \frac{\pi}{2}, 1)$ and $(-2, \frac{3\pi}{2}, 1)$ represent the same point in $\mathbb{R}^3$.

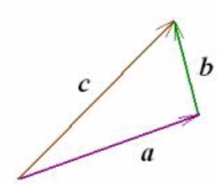
   (c) Let $\vec{u}$ and $\vec{v}$ be non-zero vectors. Then $\operatorname{proj}_{\vec{v}} \vec{u} = \vec{0}$ if and only if $\vec{u}$ and $\vec{v}$ are orthogonal.

   (d) $\vec{i} \times \vec{j} = -\vec{k}$.

   (e) There exists a vector $\vec{a}$ such that $\vec{a} \times \langle 1, 2, 3 \rangle = \langle 4, -1, 0 \rangle$.

   (f) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

   (g) In the following diagram, $\vec{b} = \vec{a} - \vec{c}$.



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