1. Evaluate the following **line integrals of scalar functions** using the definition

$$\int_C f(x,y,z)\ ds = \int_{t=a}^{t=b} f(\vec{r}(t))\|\vec{r}'(t)\|\ dt$$

where $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a parametrization of the curve C with $a \leq t \leq b$.

(a) $\int_C (x+y)\ ds$ where C is the straight-line segment from $(0,1,0)$ to $(1,0,0)$.

(b) $\int_C \sqrt{x^2+y^2}\ ds$ where C is the helix parametrized by $\vec{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ for $-2\pi \leq t \leq 2\pi$.

(c) (The answer to this problem is not zero!) Integrate $f(x,y,z) = -\sqrt{x^2+z^2}$ over the circle $\vec{r}(t) = (a\cos t)\vec{j} + (a\sin t)\vec{k}$ with $0 \leq t \leq 2\pi$

(d) $\int_C \sqrt{x+2y}\ ds$ over the straight-line segment from $(1,0)$ to $(1,2)$.

(e) $\int_C \frac{1}{x^2+y^2+1}\ ds$ where C is the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

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2. Evaluate the following **line integrals of vector fields** using the definition

$$\int_C \vec{F}\cdot d\vec{r} = \int_{t=a}^{t=b} \vec{F}(\vec{r}(t))\cdot\vec{r}'(t)\ dt$$

where $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ is a parametrization of the curve C with $a \leq t \leq b$.

(a) $\int_C \langle 3y, 2x, 4z\rangle \cdot d\vec{r}$ where C is the straight-line segment from $(0,0,0)$ to $(1,1,1)$.

(b) Integrate $\vec{F} = x^2\vec{i} - y\vec{j}$ along the curve $x = y^2$ from $(0,0)$ to $(1,1)$.

(c) Integrate $\vec{F} = x^2\vec{i} - y\vec{j}$ along the straight-line segment from $(0,0)$ to $(1,1)$.

(d) Integrate $\vec{F} = y\vec{i} - x\vec{j}$ counter-clockwise along the unit circle $x^2+y^2 = 1$ from $(1,0)$ to $(0,1)$.

(e) Integrate $\vec{F} = y\vec{i} - x\vec{j}$ along the straight-line segment from $(1,0)$ to $(0,1)$.

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3. Find the work done by the force $\vec{F} = xy\vec{i} + (y-x)\vec{j}$ as a particle moves from $(1,1)$ to $(2,3)$ along a straight line. Recall that the work W done by F over a curve C is

$$W = \int_C \vec{F}\cdot d\vec{r}$$

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4. Sometimes line integrals of vector fields are presented to you in different ways, though we always use the same method of evaluation. When we write

$$\int_C P\ dx + Q\ dy$$

for some functions P and Q, then the vector field we're integrating is $\vec{F} = \langle P, Q \rangle$. If we think of $d\vec{r} = \langle dx, dy \rangle$, then we see that

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \langle P, Q \rangle \cdot \langle dx, dy \rangle = \int_C P\ dx + Q\ dy.$$

(This is a little mathematically dubious, but gives a good justification for how the symbols fit together)

Using this, evaluate $\int_C xy\ dx + (x+y)\ dy$ where C is the curve $y = x^2$ from $(-1, 1)$ to $(2, 4)$.
