Solutions to Homework 5

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Problems Assigned

- 12.6: 28, 30, 32, 34, 36
- 13.1: 2, 6, 11, 22-24, 26, 42

12.6, #28

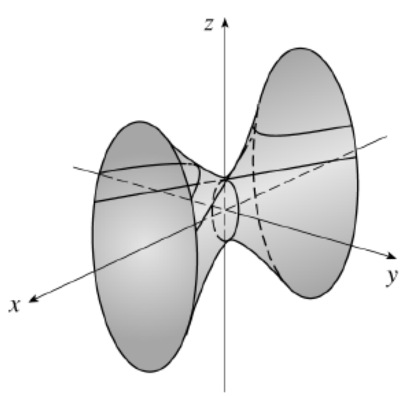
The xy-trace is the parabola $y = x^2$
The yz-trace is the parabola $y = z^2$

The only shape with these characteristics is **V.**

You can also recognize from the chart on page 837 of the textbook that this is the equation for a hyperbolic paraboloid.

12.6, #30

The elliptical traces in $x = k$ grow bigger as $\left|k\right|$ increases. This suggests that we have a hyperboloid in the x-direction. The fact that the ellipse at $k = 0$ is not a point tells us that it is a **hyperboloid of one sheet**, rather than two. The $z = k$ traces support this.



12.6, #32

Rewrite the equation in standard form.

$$4x^2 - y + 2z^2 = 0$$

$$x^2 - \frac{y}{4} + \frac{z^2}{2} = 0$$

$$x^2 + \frac{z^2}{2} = \frac{y}{4}$$

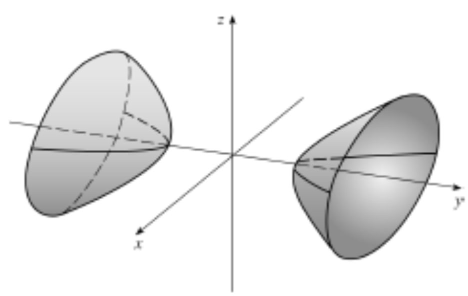
This is a **elliptic paraboloid along the *y*-axis, with vertex** $(0,0,0)$**.** See the chart on page 837 of the textbook.

12.6, #34

Rewrite the equation in standard form.

$$y^2 = x^2 + 4z^2 + 4$$
$$-x^2 + y^2 - 4z^2 = 4$$
$$-\frac{x^2}{4} + \frac{y^2}{4} - z^2 = 1$$

This is a **hyperboloid in two sheets along the *y*-axis, with vertex** $(0,0,0)$. See the chart on page 837 of the textbook.



12.6, #36

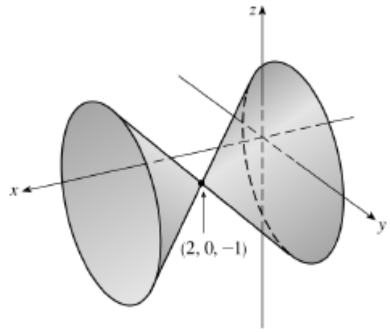
Rewrite the equation in standard form. This one is a little trickier and requires completing the square for x and z. Be especially careful with the negative sign around the z terms.

$$x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$$
$$\left(x^2 - 4x + c\right) - y^2 - \left(z^2 + 2z + d\right) - c + d + 3 = 0$$

Complete the squares and find that $c = 4, d = 1$

$$\left(x^2 - 4x + 4\right) - y^2 - \left(z^2 + 2z + 1\right) - 4 + 1 + 3 = 0$$
$$(x-2)^2 - y^2 - (z+1)^2 = 0$$
$$(x-2)^2 = y^2 + (z+1)^2$$

This is a cone in the x-direction, with vertex $(2, 0, -1)$. See the chart on page 837 of the textbook.



13.1 #2

Let us examine the domain constraints of each part of the vector.

X-Coordinate

The domain of $\cos t$ is all real numbers.

Y-Coordinate

The natural log function has a domain of $(0, \infty)$.

(To understand why, consider that $log_e x = y$ can be rewritten as $x = e^y$. Since e is a positive number, e^y has to be greater than 0 and thus, so does x .)

This gives us that $t > 0$

Z-Coordinate

The denominator cannot be equal to zero, so $t \neq 2$.

Putting it together

The constraints together give us that the domain of r is $(0, 2) \cup (2, \infty)$.

13.1 #6

X-Coordinate

Use L'Hospital

$$\lim_{t\to\infty} te^{-t} = \lim_{t\to\infty} \frac{t}{e^t} = \lim_{t\to\infty} \frac{1}{e^t} = 0$$

Y-Coordinate

Use the rules for limits of polynomials at infinity. (If you want to brush up on these, look at http://tutorial.math.lamar.edu/Classes/CalcI/LimitsAtInfinityI.aspx)

In this case, $\lim_{t\to\infty} \frac{(t^3+t)}{2t^3-1} = \lim_{t\to\infty} \frac{1+\frac{1}{t^2}}{2-\frac{1}{t^3}} = \frac{1+0}{2-0} = \frac{1}{2}$

Z-Coordinate

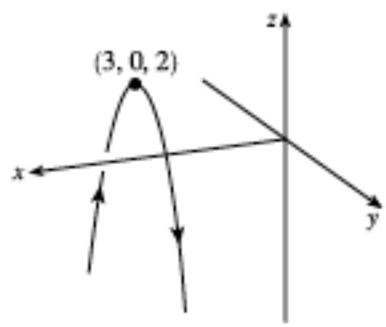
First divide the top and bottom of the fraction by $\frac{1}{t}$, then use L'Hospital's rule again.

$$\lim_{t\to\infty} t\,sin(\frac{1}{t}) = \lim_{t\to\infty} \frac{\sin\left(\frac{1}{t}\right)}{\frac{1}{t}} = \lim_{t\to\infty} \frac{\left(-\frac{1}{t^2}\right)\cos\left(\frac{1}{t}\right)}{\frac{-1}{t^2}} = \lim_{t\to\infty} \cos\left(\frac{1}{t}\right) = 1$$

The limit is $\langle 0, \frac{1}{2}, 1\rangle$

13.1 #11

The parametric equations are $x = 3$, $y = t$, $z = 2 - t^3$.
Eliminate the parameter to find $z = 2 - y^3$, and $x = 3$.
The vertex is (3, 0, 2).
Don't forget to note the direction in which t increases.



13.1 #22

Since $x = \cos(t)$, $y = \sin(t)$, all x and y points will be on the cylinder $x^2 + y^2 = 1$. The possibilities are I, IV and VI. As t increases, z approaches 0. **Therefore, it is VI.**

13.1 #24

Since $x = \cos(t)$, $y = \sin(t)$, all x and y points will be on the cylinder $x^2 + y^2 = 1$. The possibilities are I, IV and VI. Since $z = \cos 2t$, there are negative z-values and the curve repeats itself. Therefore **it must be I.**

13.1 #26

Since $x = \cos^2 t$, $y = \sin^2 t$, and $\cos^2 t + \sin^2 t = 1$, all points will be on plane where $x + y = 1$. Since x and y are periodic, and z increases with t, **this is III.**
(The graph in the textbook is hard to see.)

13.1 #42

Whenever x and y have the equation of a circle with radius r, try parametrizing using $x = r(\cos t),\ y = r(\sin t)$.

In this case, $x = 2(\cos t),\ y = 2(\sin t)$. Therefore,

$$Z = xy = 2(\cos t)2(\sin t) = 4\cos(t)\sin(t) = 2\sin(2t)$$

with the last equality following from the identity $\sin 2t = 2\sin t\cos t$.

So the parametric equations are

$$x = 2(\cos t)$$
$$y = 2(\sin t)$$
$$z = 2\sin(2t)$$

And the vector function is

$$\boldsymbol{r}(\boldsymbol{t}) = 2(\mathbf{cos}\boldsymbol{t})i + 2(\mathbf{sin}\boldsymbol{t})j + 2\mathbf{sin}(2\boldsymbol{t})k$$

With $0 \le \boldsymbol{t} \le 2\boldsymbol{\pi}$