Boonkasame Solutions

① $xy - yz + e^{xz} = 3$

(a) $\frac{\partial}{\partial x}(xy - yz + e^{xz}) = \frac{\partial}{\partial x}(3)$

$$y - y\frac{\partial z}{\partial x} + e^{xz}\left(z + x\frac{\partial z}{\partial x}\right) = 0$$

$$\frac{\partial z}{\partial x}(xe^{xz} - y) = -y - ze^{xz}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-y - ze^{xz}}{xe^{xz} - y}}$$

$\frac{\partial}{\partial y}(xy - yz + e^{xz}) = \frac{\partial}{\partial y}(3)$

$$x - z - y\cdot\frac{\partial z}{\partial y} + e^{xz}\left(\frac{\partial z}{\partial y}\cdot x\right) = 0$$

$$\frac{\partial z}{\partial y}(xe^{xz} - y) = z - x$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{z - x}{xe^{xz} - y}}$$

(b) Tangent Plane: $z = z_0 + \frac{\partial z}{\partial x}(0,-1)(x - 0) + \frac{\partial z}{\partial y}(0,-1)(y+1)$,
where z_0 is the z-coordinate when $x=0$ & $y=-1$.

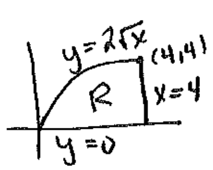
$(0)(-1) - (-1)z_0 + e^{(0)(z_0)} = 3 \rightarrow z_0 + 1 = 3 \rightarrow z_0 = 2$

$$\frac{\partial z}{\partial x}(0,-1) = \frac{-(-1) - 2e^{(0)(2)}}{(0)e^{(0)(2)} - (-1)} = \frac{1-2}{1} = -1$$

$$\frac{\partial z}{\partial y}(0,-1) = \frac{2-0}{(0)e^{(0)(2)} - (-1)} = \frac{2}{1} = 2$$

So, $\boxed{z = 2 - x + 2(y+1)}$

②



(a) Interior: $f_x = 1$, $f_y = -1$, both never 0, so no critical pts
$f(x,y)$ will be largest when x is big & y is small, so
$\boxed{\text{at } (4,0) \text{ there is a global max}}$

(b) on $x=4$: $f(4,y) = 4-y$, so $f'(4,y) = -1 \neq 0$, no global max
on $y=0$: $f(x,0) = x$, so $f'(x,0) = 1 \neq 0$, no global max
(4,0) already a global min, so all other pts on boundary are on the parabola.

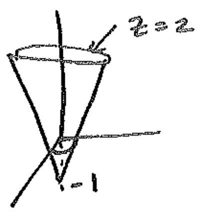
(c) on $x = y^2/4$: $f(y^2/4, y) = y^2/4 - y \rightarrow f'(y^2/4, y) = y/2 - 1 = 0 \rightarrow y = 2 \rightarrow x = 1$

(x,y)	$f(x,y) = x-y$
(1,2)	-1
(0,0)	0
(4,4)	0

so $\boxed{(1,2) \text{ a global min}}$

(3) $z = \sqrt{x^2+y^2} - 1 = 2$

$x^2+y^2 = 9$



$$\int_0^{2\pi}\int_1^3\int_{r-1}^2 \frac{1}{z+1}\cdot r\,dz\,dr\,d\theta + \int_0^{2\pi}\int_0^1\int_0^2 \frac{1}{z+1}\cdot r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^3 r\ln|z+1|\Big|_{r-1}^2\,dr\,d\theta + \int_0^{2\pi}\int_0^1 \ln|z+1|\Big|_0^2\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^3 r\ln 3 - r\ln r\,dr\,d\theta + \int_0^{2\pi}\int_0^1 \ln 3 - \ln 1\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} \ln 3\cdot\frac{r^2}{2}\Big|_1^3 d\theta - \int_0^{2\pi}\int_1^3 \underbrace{r}_{dv}\underbrace{\ln r}_{u}\,dr\,d\theta + \int_0^{2\pi} \ln 3\cdot \frac{r^2}{2}\Big|_0^1 d\theta$$

$$= \int_0^{2\pi}\left(\frac{9}{2}\cdot\ln 3 - \frac{1}{2}\ln 3\right)d\theta - \int_0^{2\pi}\left[(\ln r)\left(\tfrac{1}{2}r^2\right)\Big|_1^3 - \int_1^3 \tfrac{1}{2}r^2\cdot\tfrac{1}{r}\,dr\right]d\theta + \int_0^{2\pi}\tfrac{1}{2}\ln 3\,d\theta$$

$$= 2\pi\cdot 4\ln 3 - \int_0^{2\pi} \ln 3\cdot\frac{9}{2} - \frac{1}{2}\cdot\left(\frac{r^2}{2}\right)\Big|_1^3 d\theta + \pi\ln 3$$

$$= 9\pi\ln 3 - 9\pi\ln 3 + \int_0^{2\pi}\frac{9}{4} - \frac{1}{4}\,d\theta$$

$$= \boxed{4\pi}$$

(4) $(1,3,-1)$ & $(2,1,1)$, $g(x,y,z) = x(y+z)$

ave value of $g = \dfrac{1}{\text{length of wire}}\cdot\int_C g(x,y,z)\,ds$

length of wire $= \sqrt{1^2+2^2+2^2} = \boxed{3}$

$C: \vec{x}(t) = \langle 1,3,-1\rangle + t\langle 1,-2,2\rangle = \langle 1+t,\ 3-2t,\ 2t-1\rangle$

$0 \le t \le 1$ $\qquad \vec{x}\,'(t) = \langle 1,-2,2\rangle,\ \|\vec{x}\,'(t)\| = \sqrt{1+4+4} = 3$

$$\int_C g\,ds = \int_0^1 (1+t)(3-2t+2t-1)\cdot 3\,dt$$

$$= \int_0^1 (1+t)\cdot 6\,dt = 6\left[t+\frac{t^2}{2}\right]_0^1 = 6(3/2) = \boxed{9}$$

So, average value $= 9/3 = \boxed{3}$

(5b) $C: \vec{x}(t) = \langle \cos t, \sin t\rangle$

$0 \le t \le 2\pi$

$\vec{x}\,'(t) = \langle -\sin t, \cos t\rangle$

$$\int_0^{2\pi}\left\langle \frac{-\sin t}{\cos^2 t+\sin^2 t}, \frac{\cos t}{\cos^2 t+\sin^2 t}\right\rangle\cdot\langle -\sin t, \cos t\rangle\,dt$$

$$= \int_0^{2\pi}\langle -\sin t, \cos t\rangle\cdot\langle -\sin t, \cos t\rangle\,dt$$

$$= \int_0^{2\pi}\sin^2 t + \cos^2 t\,dt = \int_0^{2\pi} dt = \boxed{2\pi}$$

⑥ $\vec{F}(x,y,z) = \langle \underbrace{y(z-\sin(xy))}_{P}, \underbrace{x(z-\sin(xy))}_{Q}, \underbrace{xy}_{R} \rangle$

(a) $\int P\,dx = xyz + \cos(xy) + C(y,z)$

$\int Q\,dy = xyz + \cos(xy) + D(x,z)$

$\int R\,dz = xyz + E(x,y)$

so $\boxed{f(x,y,z) = xyz + \cos(xy)}$

(b) $(0,0,0)$ to $(1,\pi,-1)$. By the FTC of line integrals,

$$\int_C \vec{F}\cdot d\vec{x} = f(1,\pi,-1) - f(0,0,0) = (-\pi + \cos\pi) - (0 + \cos(0))$$

$$= -\pi - 1 - 1$$

$$= \boxed{-\pi - 2}$$

⑩ $\rho = \sqrt{x^2+y^2+z^2}$

(a) $\frac{\partial \rho}{\partial x} = \frac{2x \cdot \frac{1}{2}}{\sqrt{x^2+y^2+z^2}} = \frac{x}{\rho}$ (& similarly for $\frac{\partial \rho}{\partial y}$ & $\frac{\partial \rho}{\partial z}$).

(b) $\vec{F}(x,y,z) = \langle \rho^2 x, \rho^2 y, \rho^2 z \rangle$

$$\vec{\nabla}\cdot\vec{F} = \langle \partial/\partial x, \partial/\partial y, \partial/\partial z \rangle \cdot \langle \rho^2 x, \rho^2 y, \rho^2 z \rangle$$

$$= \frac{\partial}{\partial x}(\rho^2 x) + \frac{\partial}{\partial y}(\rho^2 y) + \frac{\partial}{\partial z}(\rho^2 z)$$

$$= 2\rho \cdot \frac{\partial \rho}{\partial x} \cdot x + \rho^2 + 2\rho \frac{\partial \rho}{\partial y} \cdot y + \rho^2 + 2\rho \frac{\partial \rho}{\partial z} \cdot z + \rho^2$$

$$= 3\rho^2 + 2\rho\left(\frac{x}{\rho}\right)\cdot x + 2\rho\left(\frac{y}{\rho}\right) y + 2\rho\left(\frac{z}{\rho}\right)\cdot z$$

$$\boxed{\vec{\nabla}\cdot\vec{F} = 3\rho^2 + 2x^2 + 2y^2 + 2z^2 = 3\rho^2 + 2\rho^2 = 5\rho^2}$$