Problems from Section, Thursday 9/10

1.

$$\int \sin^6 x \cos^3 x \, dx = \int \sin^6 x \cos^2 x \cos x \, dx$$

$$= \int \sin^6 x (1 - \sin^2 x) \cos x \, dx \qquad u = \sin x, \, du = \cos x \, dx$$

$$= \int u^6 (1 - u^2) \, du$$

$$= \int (u^6 - u^8) \, du$$

$$= \frac{1}{7} u^7 - \frac{1}{9} u^9 + C$$

$$= \frac{1}{7} \sin^7 x = \frac{1}{9} \sin^9 x + C$$

2.

$$\int \sin(1/2x) \cos(1/2x) \, dx = \frac{1}{2} \int \sin x \, dx$$

$$= -\frac{1}{2} \cos x + C$$

or

$$\int \sin(1/2x) \cos(1/2x) \, dx = 2 \int u \, du, \quad u = \sin(1/2x), \, du = \frac{1}{2} \cos(1/2x) \, dx$$

$$= 2 \sin^2(1/2x) + C$$

3.

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx, \quad u = \cos x, \, du = -\sin x \, dx$$

$$= -\int \frac{1}{u} \, du$$

$$= -\ln \|u\| + C$$

$$= -\ln |\cos x| + C$$

4.

$$\int \sin^4 x \, dx = \int (\sin^2 x)^2 \, dx$$

$$= \int \left(\frac{1 - \cos(2x)}{2} \right)^2 dx$$

$$= \frac{1}{4} \int (1 - 2\cos(2x) + \cos^2(2x)) \, dx$$

$$= \frac{1}{4} \int \left(1 - 2\cos(2x) + \frac{1}{2}(1 + \cos(4x)) \right) dx$$

$$= \frac{1}{4} \left(x - \sin(2x) + \frac{1}{2} \left(x + \frac{1}{4}\sin(4x) \right) \right) + C$$

$$= \frac{1}{32}(12x - 8\sin(2x) + \sin(4x))$$